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NOTES ON ELLIPTICITY IN ECLIPSING BINARIES

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ABSTRACT

1. The constant K, derived from Cowling's formula for apsidal motion in a close binary, is computed for 19 systems. Many of the values are rough, but all are small. The largest value found from reliable data is 0.018—very much nearer to a Roche model (0.000) than to a homogeneous one (0.750). Refinements of calculation would diminish the calculated values. The values of K certainly differ from star to star.

2. The "photometric ellipticity" derived from the variation outside eclipse depends on the darkening at the limb and the variation of surface brightness with gravity, as well as on the geometrical form. The gravity effect approximately doubles the observed variation (eq. [14]). The gravity effect can be well studied by this means; the geometrical shape and internal constitution cannot be found in this way.

3. The light-curve resulting from an eclipse of two prolate, uniformly bright, ellipsoidal stars of different shapes can be very closely represented by the eclipse of two stars of the same shape but of different dimensions. The outstanding residuals simulate the effect of a small degree of darkening at the limb. It is impracticable to find the difference in shape from the light-curve.

4. It is recommended that the ellipticity of figure calculated on dynamical principles from the Roche model (or perhaps with $K = 0.01$) should be used in determining the geometrical elements of eclipsing variables.

I. APSIDAL MOTIONS AND CENTRAL CONDENSATION

1. The theory of apsidal motion in close binaries appears to have been cleared up by the recent work of Cowling[1] and Sterne.[2] Walter[3] now agrees with their conclusion that for gaseous, and therefore almost perfectly fluid, bodies, the figure at any moment will be very nearly that of equilibrium under the instantaneous tidal forces.

[1] *M.N.*, **98**, 734, 1938.

[2] *M.N.*, **99**, 451, 1939.

[3] *Zs. f. Ap.*, **17**, 320, 1939. For solid bodies, Walter's former conclusion that the apse regresses is correct.

The periastron advances. The principal term in its mean motion, n', is

$$\frac{n'}{n} = K_1 \left(\frac{R_1}{r}\right)^5 \left(1 + 16\,\frac{m_2}{m_1}\right) + K_2 \left(\frac{R_2}{r}\right)^5 \left(1 + 16\,\frac{m_1}{m_2}\right), \qquad (1)$$

where n is the orbital mean motion, R_1 and R_2 the radii of the stars, r that of the orbit, m_1 and m_2 the masses, and K_1 and K_2 constants, depending on the density distribution, which have the value 0.750 for homogeneous stars and 0.000 for the "Roche model" with all the mass at the center. Terms of higher order in R/r and in the eccentricity increase the motion. If the mass ratio, radii, and apsidal motion are known and we set $K_1 = K_2$ in (1), we obtain a mean value of K which gives information regarding the internal constitution.

2. Of the observational data, m_1/m_2 is rarely directly determinable. Unless R_1 and R_2 are decidedly different, a rough value will suffice, and this may be obtained from the relative brightness (assuming, for example, that m varies as $L^{1/3}$).

Errors in R_1 and R_2 are much more serious. Owing to observational selection (see below), the systems in which apsidal motion has been established show primary and secondary eclipses of comparable depth, rarely exceeding $0^m.6$. These are just the conditions under which the determination of the relative radii of the stars from the photometric data alone is practically indeterminate, even with a good light-curve. The ratio of the surface brightness is determined, and also the sum of the radii, from the duration of the eclipses (so long as these are not too shallow).

As a typical case, we may take the depths of minima equal, and the sum of the radii as $2R$, and set $R_1 = R(1 + a)$ and $R_2 = R(1 - a)$ We then have

$$\frac{m_1}{m_2} = \left\{ \frac{(1 + a)}{1 - a} \right\}^{2/3};$$

and we find, for the coefficient C of KR^5/r^5 in equation (1):

a	0.0	0.1	0.2	0.3	0.4
R_1/R_2	1.00	1.22	1.50	1.86	2.33
C	34.0	35.5	40.0	47.2	56.7
$(C/34)^{1/5}$	1.000	1.009	1.032	1.068	1.108

The last line gives the factor by which R must be multiplied to give the same value of K for equal stars. A solution assuming equal radii will give the maximum value of K. This will be a good approximation unless the radii are decidedly unequal.

3. Unless the light-curve is really poor, the duration of the eclipse at half the maximum depth can be found from it with an uncertainty of a few per cent. The whole duration, between the unobservable outer contacts, must be calculated from this. If θ_0 and $\theta_{\frac{1}{2}}$ are the longitudes (assuming a circular orbit) corresponding to the first contact and half the maximum loss of light, we have[4]

$$\frac{\sin^2 \theta_0}{\sin^2 \theta_{\frac{1}{2}}} = \chi(k, \alpha_0\ 0) ,$$

where k is the ratio of the radii and α_0 measures the maximum eclipse. This function has different values for uniform disks, for darkened disks with the small star behind, and for the same with the small star in front.[5] Call them χ_0, χ_0', and χ_0''. If $1 - \lambda_1$ is the depth of the minimum with the small star behind, and $1 - \lambda_2$ that of the other, we have for uniform disks

$$\alpha_0 = 1 - \lambda_1 + \frac{1 - \lambda_2}{k^2} .$$

Assuming various values of α_0, k and χ_0 may be found. For darkened disks we have

$$\alpha_0' = 1 - \lambda_1 + \frac{1 - \lambda_2}{Q(k, \alpha_0')} ; \qquad \alpha_0''Q(k, 1) = \alpha_0'Q(k, \alpha_0') .$$

With any assumed value of α_0', the values of k, α_0'', χ_0', and χ_0'' may be found with the aid of tables.[5]

For typical cases in which the minima are of equal depth we find the results given in Table 1.

For uniform disks all the solutions give nearly the same light-curve and the same computed duration of eclipse. When the eclipse approaches internal tangency of the disks, χ_0 becomes smaller, which diminishes the computed length and counteracts the effect discussed

[4] Russell, *Ap. J.*, **35**, 326, 1912. [5] *Ap. J.*, **36**, 392–394, 1912.

above. In this case, however, the eclipse chord may be considerably less than the sum of the diameters of the disks, the computed radii will be greater, and K will be smaller.

For darkened disks, if the stars are unequal, the durations of the two minima at half-depth will be decidedly different, though the orbit is circular and the extreme widths are the same. The average value of χ_0 for the two minima is less than for uniform disks for the deepest eclipses, but is greater for the shallower ones.

TABLE 1

HALF-WIDTHS OF ECLIPSES

Depth $(1-\lambda_1)$	Uniform			Darkened				
	a_0	k	χ_0	a_0'	k	a_0''	χ_0'	χ_0''
0.50	1.00	1.00	6.15	1.00	1.00	1.00	5.44	5.44
0.40	0.80	1.00	4.43	0.80	1.00	0.800	4.64	4.64
	0.90	0.895	4.47	0.90	0.865	0.840	4.43	4.81
	1.00	0.817	4.43	1.00	0.789	1.000	3.80	5.00
0.30	0.60	1.000	3.76	0.60	1.000	0.600	4.24	4.24
	0.70	0.866	3.74	0.70	0.856	0.625	4.12	4.34
	0.80	0.755	3.73	0.80	0.766	0.693	3.98	4.39
	0.90	0.707	3.69	0.90	0.687	0.795	3.74	4.40
	1.00	0.655	3.54	1.00	0.601	1.000	3.27	4.44
0.20	0.40	1.000	3.30	0.40	1.000	0.400	3.94	3.94
	0.60	0.707	3.31	0.60	0.720	0.486	3.76	4.05
	0.80	0.578	3.26	0.80	0.586	0.656	3.55	4.05
	0.90	0.535	3.22	0.90	0.535	0.772	3.34	4.02
	1.00	0.500	3.00	1.00	0.467	1.000	2.82	3.98

It follows from this survey that in almost all the cases in which apsidal motions have thus far been observed the assumption of uniform disks of equal size will lead to a maximum value of the constant K in equation (1).

In systems of a "typical Algol" character, with a large faint companion of small mass, almost the whole of the apsidal motion arises from the ellipticity of the latter, and more detailed knowledge of the elements is necessary.

4. The observational determination of the apsidal motion is more delicate. It is never safe to assume that a periodic term in the time

of primary minimum is due to this cause, for light-equation arising from motion of the close pair in a large orbit can produce almost exactly the same effect. Moreover, many eclipsing pairs show more complicated irregularities, which are obviously not simply periodic and are thus far unexplained.[6] Only when the secondary minimum is observable and shows displacements in the opposite sense to the primary can apsidal motion safely be inferred. The absence of stars with shallow secondaries from our present lists is thus natural.

In a few cases a complete revolution of the apsides, or more, has been observed, and the motion is well determined. When both the interval and the widths of the two minima may be well determined from the observations, and also the rate of change of the interval, a direct determination of n' may be made, as in the case of *HV* 7498.[7] Considerable new information is available for other cases, which are therefore discussed in detail.[8]

From the interval of minima we have

$$\Delta = t_2 - t_1 - \tfrac{1}{2}P = \frac{2P}{\pi} e \cos (\omega_0 + n't)$$

$$= \frac{2P}{\pi} \{e \cos \omega_0 (1 - \tfrac{1}{2}n'^2t^2) - n't\, e \sin \omega_0\} .$$

If only the times of minima are available, we must wait until the curvature of the plot of Δ against t becomes sensible before n' can be found; but it is possible to set a minimum for n', namely, the value which gives the greatest curvature consistent with the observations.

a) For example, the Harvard observations of RU Monocerotis[9] ($P = 3^{\mathrm{d}}.5847$) give the intervals between minima as shown in Table 2. The residuals O − C are the departures of ω from uniform motion at the rate given below. For $e = 0.40$ they are inadmissibly great; they are tolerable for $e = 0.45$ or greater. A period of 600 years ($n' = 0^\circ.6$) is about the minimum permissible and will give the greatest permissible value of K.

[6] R. S. Dugan and Frances W. Wright, *Princeton Contr.*, No. 19 (in press).

[7] M. B. Shapley, unpublished.

[8] An independent discussion of some of these has been made by Sterne, *M.N.*, **99**, 662, 1939.

[9] Shapley, *Proc. Amer. Acad. Arts and Sciences*, **66**, 469, 1931 (Harvard Reprint No. 72).

When (as in this case) $\cos i$ is small compared with R, the durations of the two eclipses are approximately in the ratio $(1 + e \sin \omega)/(1 - e \sin \omega)$, or 1.7 for $e \sin \omega = 0.26$. Though, as Shapley says, "the widths of the minima cannot be estimated with sufficient precision to give a value of $e \sin \omega$," his light-curve is hardly consistent with so great a difference as this. For unequal darkened disks with the small star in front at the eclipse nearer periastron the ratio might be reduced by about 12 per cent (Table 1), but it would still be at the limit of the admissible, and the period of 600 years may be adopted.

TABLE 2

APSIDAL MOTION IN RU MONOCEROTIS

T	t_2-t_1	$e \cos \omega$	$e=0.40$		$e=0.45$		$e=0.50$	
			ω	O−C	ω	O−C	ω	O−C
1902.0	2^d69	0.394	10°0	−2°6	29°0	−0°9	38°0	−0°6
1907.0	2.65	.377	19.6	+2.2	33.2	+0.5	41.1	+0.4
1914.0	2.60	.355	27.4	+3.0	38.0	+1.5	44.8	+1.2
1925.1	2.56	0.337	32.6	−2.6	41.5	−1.1	47.6	−0.7
n'			0°98		0°55		0°42	
$e \sin \omega$			0.127		0.262		0.341	

With so large an eccentricity spectroscopic observations would settle the question—and are much to be desired.

b) Miss Jones's careful study of V523 Sagittarii[10] gives the following normals:

Date	$e \cos \omega$	O−C₁	O−C₂	O−C₃
1899.8	+0.011	−0.004	0.000	0.000
1911.5	− .060	+ .004	+ .001	+ .001
1923.8	− .107	+ .001	− .003	− .003
1934.0	−0.143	−0.006	−0.001	+0.003

From these, with various assumptions regarding the eccentricity, we find

(1) $e = 0.15$; $\omega = 90° + 2°10\,(t - 1902.5)$; period = 171 years
(2) .20 $1.41\,(t - 1902.0)$ 255
(3) 0.25 $1.12\,(t - 1901.0)$ 322

[10] *Harvard Bull.*, No. 909, 10, 1938.

The first period appears to be a little too short; the other two give an excellent fit.

Good mean light-curves are available for both minima. The one which comes late is narrower, showing that the periastron is advancing. A uniform solution with equal stars, and $e \sin \omega = 0.15$, gives a satisfactory representation of both, and the elements $R_1 = R_2 = 0.222$, $i = 85.0^\circ$, $L_1 = L_2 = 0.50$. This curve should correspond closely to the mean value $e \cos \omega = -0.08$, and hence to $e = 0.17$.

Any considerable increase in $e \sin \omega$ gives a greater difference in the computed widths of the minima than the observations permit. The values $e = 0.17$, $P' = 200$ years, may therefore be adopted with some confidence—though for unequal darkened disks $e = 0.25$ might be permissible.

c) For YY Sagittarii[11] we find, in the same manner,

Date	$e \cos \omega$	$O-C_1$	$O-C_2$	$O-C_3$
1898.3	−0.047	−0.004	−0.002	0.000
1911.3	.072	+ .004	+ .001	+ .001
1924.3	.103	+ .007	+ .002	+ .001
1937.3	−0.135	−0.001	−0.002	−0.001

(1)	$e = 0.15$;	$\omega = 129.1^\circ + 1.27^\circ\ (t - 1917.8)$;	$P' = 285$ years
(2)	.20	$116.5 + 0.785$	460
(3)	0.30	$107.2 + 0.476$	757

The observers state that "no certain difference is now found in the widths of the two minima, though about 1880 when ω was 90°, there was probably a measurable difference." This is hardly compatible with $e = 0.20$, which makes the ratio of the widths 1.5. We may adopt $e = 0.17$, $P' = 350$ years.

d) Of the two stars observed by Dugan and Miss Wright,[12] V526 Sagittarii shows a change in $e \cos \omega$ from -0.18 to $+0.14$ in $7000P$, or 36.7 years. With $e = 0.20, 0.25, 0.30$, $P' = 125, 162, 205$ years. The ratio of widths of the minima is about 1.3 according to these observations, and 1.4 according to O'Connell.[13] We may adopt $e = 0.23$, $P' = 150$ years.

[11] Shapley and Miss Swope, *Harvard Bull.*, No. 909, p. 9.

[12] *Princeton Contr.*, No. 19 (in press).

[13] *Riverview Pub.*, **1**, 13, 1935.

For V346 Centauri the range in $e \cos \omega$ is -0.12 to $+0.09$ in $2250P = 38.9$ years. With $e = 0.15$, 0.20, $P' = 160$, 223 years. From the rather discordant observations the minima do not differ much in width, so that we may adopt the first.

e) The observations of GN Normae show that $e \cos \omega = +0.16 + 0.0018t$ (t in years). The widths of the minima are given by Kruytbosch[14] as $0.15P$ and $0.14P$, so that $e \sin \omega$ appears to be small and negative. The data are rough and partly inconsistent but may be represented by $e = 0.25$, $P' = 680$ years.

f) Finally, there are two systems which show a displacement of the secondary, without sensible change. For SS Lacertae ($P = 14^{d}42$) the motion may be expected to be very slow (sec. 8). KT Centauri ($P = 4^{d}13$) shows $e \cos \omega = +0.20$—changing by probably less than 0.02 in 40 years. According to Uitterdijk,[15] one minimum is 0.82 as wide as the other, which would make $e \sin \omega$ 0.1 and $P' > 1200$ years. But this difference in width is no more than could be produced by limb darkening of unequal stars with $e \sin \omega = 0$. In this case, ω might have ranged from 335° to 25°, while $e \cos \omega$ remained between 0.19 and 0.21, which would give $P' > 290$ years.

g) For RZ Cassiopeiae a small periodic displacement of the primary minimum, of period 18 years, has been detected by De Sitter.[16] This corresponds to an eccentricity 0.013, too small to be detected spectroscopically; and the secondary is too shallow to determine whether it oscillates in opposite phase. This case has been included for completeness.

In a few instances it is possible to determine the apsidal motion from radial velocities. Luyten[17] has shown that most of the motions previously announced are illusory. Advance is probable for α Virginis and 13 Ceti A. In 29 Canis Majoris ω was the same in 1916 and 1935, but the possibility of two complete revolutions in the 19 years cannot be excluded. The periastron of AO Cassiopeiae appears to make half a revolution (or perhaps $1\frac{1}{2}$ or $2\frac{1}{2}$) in 8.1 years.

5. The data, as finally collected for 18 systems, are summarized

[14] *B.A.N.*, **6**, 233, 1932.

[15] *B.A.N.*, **6**, 247, 1932.

[16] *B.A.N.*, **7**, 119, 1933.

[17] *Pub. Minnesota Obs.*, **2**, 29, 1935.

in Table 3. They are listed approximately in the order of reliability, with the best at the top. The values of m_2/m_1 given in parentheses have been estimated from the relative brightness; those in square brackets are rough guesses—whose error, however, will not seriously affect the results (see sec. 2). The densities ρ_1 and ρ_2 are in grams per cubic centimeter, P' is the apsidal and P the orbital period, and C is the coefficient of K in equation (1). The references in the last column give the source of the orbital elements.

6. For the last four stars the apsidal motion has been determined by Luyten from spectroscopic data.[18] The determination of R/r is good for 29 Canis Majoris, poor for AO Cassiopeiae, and very poor for α Virginis. For 13 Ceti A there are no eclipses, but approximate values of these ratios may be obtained indirectly. The close pair is a component of a visual binary. Let the masses of the principal component and the spectroscopic and the visual companions be μ, μx_1, and μx_2. Then

$$\mu(1 + x_1 + x_2) = a''^3 Y^{-2} p^{-3} ,$$

$$\mu(1 + x_1) = C r^3 D^{-2} ,$$

where Y is the visual period in years, a'' the apparent mean distance, p the parallax, D the spectroscopic period in days, r the corresponding mean distance in solar radii, and $C = 1.34 \times 10^{-2}$. If L is the visual luminosity, J the surface brightness, and R the radius, in solar units,

$$L = JR^2 = lp^{-2} ,$$

where $\log l = -0.4\,(m + 0.15)$ (m being the star's visual magnitude, and -0.15 the sun's magnitude for $p = 1''$). Eliminating among these equations, we find

$$\frac{R}{r} = \frac{l^{1/2}}{a''}\left(\frac{Y}{D}\right)^{2/3}\left(\frac{1 + x_1 + x_2}{1 + x_1}\right)^{1/3} C^{1/3} J^{-1/2} .$$

All the factors here are known except J, which may be estimated from the effective temperature.

[18] *Ibid.*

TABLE 3

BINARIES WITH MOVING APSIDES

Star	Mag.	Sp.	P	R_1/r	R_2/r	m_1/m_2	ρ_1	ρ_2	e	$P'/1000P$	1000C	K	Ref.
Y Cyg....	6.9	O9.5	2.996	0.206	0.206	1.01	0.12	0.12	0.14	5.6	12.6	0.014	1
GL Car....	8.9	Bp	2.422	.215	.215	[1]	0.16	0.16	.16	3.8	15.6	.017	2
AG Per.....	6.6	B3	2.029	.263	.184	1.13	0.13	0.35	.05	9.6	23	.0045	3
CO Lac....	10.2	F	1.542	.24	.24	[1]	0.29	0.29	.03	9.5	27	.004	4
V523 Sgr....	10.1	A5	2.324	.22	.22	[1]	0.16	0.16	.17	31	18	.0018	5
HV 7498....	10.2	A0	3.471	.123	.147	(1.08)	0.44	0.24	.55	63	1.7	.009	6
YY Sgr.....	9.8	A0	2.628	.127	.127	[1]	0.67	0.67	.17	49	1.1	.018	6
V526 Sgr....	9.9	A0	1.919	.12:	.12:	[1]	1.5	1.5	.23	28	0.85	.042:	7
V346 Cen...	8.3	B5	6.322	.17:	.17:	[1]	0.05	0.05	.15	9	5	.022:	7
RU Mon...	10.5	A5	3.584	.13	.11	[1]	0.34	0.55	.44	61	0.9	.018	8
V380 Cyg...	5.7	B2	12.426	.347	.125	1.33	0.0017	0.027	.22	8:	65	.002:	9
MR Cyg....	8.5	A0	1.677	.295	.219	1.18	0.14	0.29	.12	14:	42	.0017:	10
GN Nor....	12.7	....	5.703	.22:	.22:	[1]	0.03:	0.03:	.25	44	17	.0013:	11
KT Cen....	12.0	....	4.130	.14	.14	[1]	0.20	0.20	.22 .20	>100 > 25	2	< .005 < .020	12
SS Lac....	10.3	A	14.416	.084	.057	(1.3)	0.09	0.22	> .11	....	0.06	....	13
RZ Cas....	6.4	A2	1.195	.225	.295	(2.7)	0.85	0.14	.013?	5.5?	103	.002?	14
29 CMa...	4.9	O8.5	4.393	.38	.30	1.33	0.010	0.016	.1	0.8?	161	.008?	15
α Vir.....	1.2	B2	4.014	.3?	.25?	1.55	0.03?	0.03?	.10	10	52?	.002?	16
AO Cas....	6.1	O8.5	3.523	.55:	.40:	1.12	0.005:	0.011:	.04	1.7?	970?	.0006?	17
13 Cet A..	5.7	G0	2.082	0.18:	0.07?	3:	0.55:	3.2?	0.1	10:	1.3	0.08:	18

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16. Gaposchkin, *Veröff. U. Sternwarte Berlin-Babelsberg*, **9**, Part V, 76, 1932.
17. Pearce, modified by Kuiper, *Ap. J.*, **88**, 501, 1938.
18. See below.

For 13 Ceti, $a'' = 0.244$, $V = 6.91$, $m_1 = 5.7$, whence $l = 0.0046$: Luyten's discussion[19] gives $m_1 = 0.95$, $m_2 = 0.32$, $m_3 = 1.05$, whence $x_1 = 0.34$, $x_2 = 1.1$. Then $R/r = 0.178J^{-1/2}$. The Harvard spectrum is G0, whence $J = 1$. The spectroscopic companion is probably an M dwarf as Luyten suggests, and its radius about half the sun's. The separation of the close pair for mass 1.27 is 7.4 solar radii, and $R_2/r = 0.07$. These are the tabular values. The Mount Wilson spectrum is F7, corresponding to a temperature about 330° higher, on Kuiper's scale, and $J = 1.25$, which makes $R_1 = 0.16r$, $K = 0.13$.

The apsidal motion is uncertain, owing to unexplained discordances in the radial velocities. New spectroscopic observations are badly needed.

7. The principal result of this investigation is that the constant K is always small. Only in one case is the computed value as much as one-tenth as great as for a homogeneous body, and this value is very uncertain. No value derived from the better observations exceeds 0.02.

This cannot be a result of observational selection, for rapid apsidal motions are easier to detect, and these lead to large values of K. The approximations made in the discussion (assuming the stars equal, and taking the shortest permissible apsidal period) also increase the computed K. The more complete theory (which Sterne has applied to those cases which warrant it) also leads to a smaller value of K than the present first approximation.

The number of cases now available is clearly great enough to be statistically significant. There is, therefore, very strong evidence that the *density concentration within the stars is high.*

It is also evident that the stars are not all built on the same model. Conclusive proof is afforded by GL Carinae and V523 Sagittarii, for which the periods and radii are nearly the same, while the apsidal periods are 25 and 200 years.[20]

8. The value of K corresponding to any given density model may be found by integrating Radau's differential equation by quadratures. For polytropic spheres Chandrasekhar finds the values given in Table 4. The ratio of the central to the mean density from $n = 1$

[19] *Ap. J.*, **78**, 225, 1933.

[20] The same conclusion has been reached in a more detailed discussion by Sterne in *M.N.*, **99**, 662, 1939 (kindly communicated in manuscript).

to $n = 4$ is given within 5 per cent by the empirical approximation $\rho_c/\bar{\rho} = 0.825/K$.

For the stars just mentioned, the values of n are 2.9 and 3.9, corresponding to 48 and 460 for $\rho_c/\bar{\rho}$. Those for the other well-determined cases lie between these limits. The lowest value for a poorly observed case is $n = 2.0$, $\rho_c/\bar{\rho} = 11$.

TABLE 4

n	0	1	1.5	2	3	4
K	0.750	0.260	0.1446	0.0741	0.0144	0.00134
$\rho_c/\bar{\rho}$	1.000	3.290	5.991	11.403	54.18	622.41
$k\rho_c/\bar{\rho}$	0.750	0.853	0.864	0.845	0.782	0.823

The two cases of apparently "fixed" apsides in eclipsing pairs are explicable by values of K within the same limits. For SS Lacertae, if $K = 0.02$, the apsidal period comes out 33,000 years. The spectroscopic data are consistent (within their present wide uncertainty) with similar values of K. Agreement for AO Cassiopeiae may be secured by assuming that the periastron has made $1\frac{1}{2}$ or $2\frac{1}{2}$ revolutions instead of $\frac{1}{2}$ revolution.

Further photometric and spectroscopic observations of these stars and of any others which show evidence of eccentric orbits are urgently needed. Until they have been obtained, the question why apsidal motion has been so hard to detect in pairs where the components are nearly in contact must remain unanswered. It may be noted, however, that for some of the "good" systems in Table 3 the interval between the nearest points of the stars hardly exceeds their diameters.

II. PHOTOMETRIC AND DYNAMICAL ELLIPTICITY

It has been maintained that the photometric observations of eclipsing pairs outside eclipse show ellipticities which indicate that the stars are nearly homogeneous. The relation between this "photometric ellipticity" and the actual ellipticity of figure has been discussed by Walter[21] and recently by Luyten,[22] but something more may be added.

[21] *Schriften der Königsberger Gelehrten Gesellschaft*, **3**, 55–100, 1931.

[22] *M.N.*, **98**, 459, 1938.

There is general agreement on the forms which are to be expected for a pair of stars revolving in a circular orbit and rotating with the same period—at least, to the first order in the ellipticity (which alone will be considered in this paper). Let m_1 and m_2 be the masses; take the radius of the orbit as unit, and let a_1, b_1, and c_1 be the semi-axes of star 1. Set

$$a_1 = b_1(1 + u_1) ; \qquad c_1 = b_1(1 - v_1) . \tag{2}$$

The ratio of centrifugal force to gravity at the equator is

$$\varphi_1 = \frac{m_1 + m_2}{m_1} a_1^3 .$$

Then

$$\left.\begin{aligned} u_1 &= \frac{3}{2}\frac{m_2}{m_1} a_1^3(1 + 2K_1) , \\ v_1 &= \frac{1}{2}\frac{m_1 + m_2}{m_1} a_1^3(1 + 2K_1) , \end{aligned}\right\} \tag{3}$$

where K_1 is the parameter depending on the internal constitution, already defined—and similarly for star 2.

The light-changes which arise from the rotation of an ellipsoid depend upon the degree of darkening at the limb and the degree to which the surface brightness J varies with the "local" gravity g.[23] According to von Zeipel[24] and Chandrasekhar,[25] J should be proportional to g; but the proof of this depends on the assumption that there is no internal circulation of matter; and, in any case, it applies to the total radiation. Changes in radiation of a given wave length may be greater or less.[26] We will therefore set, for the apparent surface brightness of an element of the disk, for which the normal is inclined at an angle γ to the line of sight

$$J \doteq J_0(1 - x + x\cos\gamma)\left(1 - y + \frac{yg}{g_0}\right) , \tag{4}$$

[23] Takeda, *Mem. College Sci., Kyoto Imperial U.*, A, **17**, 197, 1934.

[24] *M.N.*, **84**, 665, 1924.

[25] *M.N.*, **93**, 573, 1933.

[26] Cf. Kopal, *Ap. J.*, **89**, 323, 1939.

where g_0 is the mean value of gravity over the surface. If, as is probable, the radiation is nearly of black-body quality, the value of y should increase for the shorter wave lengths. The light received from the star is, then,

$$L = \int J \cos \gamma d\sigma \tag{5}$$

over the visible half of the ellipsoid.

If A, B, and C are the amounts of light sent in the directions of the axes a, b, and c, then, to the first order in the ellipticities, the light sent in any direction will be

$$L = C \sin^2 \delta + B \cos^2 \delta \cos^2 t + A \cos^2 \delta \sin^2 t ,$$

where the direction is toward declination δ and hour angle t, for an observer at the end of axis b.

When $x = y = 0$ (uniform disks), $A:B:C = bc:ca:ab$; and we have

$$L_1 = \pi b^2 J_0 (1 + u - v + v \sin^2 \delta - u \cos^2 \delta \cos^2 t) . \tag{6}$$

When $x = 1$, $y = 0$ (complete darkening),[27]

$$L_2 = \tfrac{2}{3}\pi b^2 J_0 \{1 + \tfrac{8}{5}(u - v) + \tfrac{8}{5}(v \sin^2 \delta - u \cos^2 \delta \cos^2 t)\} . \tag{7}$$

By Clairaut's Theorem[28] the variation in gravity from pole to equator on a spheroid is

$$\frac{\Delta g}{g} = -\tfrac{5}{2}\varphi + \epsilon ,$$

where ϵ is the oblateness. But $\epsilon = \frac{1}{2}\varphi(1 + 2K)$. It follows that at any point on the ellipsoid

$$\frac{g - g_0}{g_0} = \frac{2K - 4}{2K + 1} \cdot \frac{r - r_0}{r_0} , \tag{8}$$

[27] *Ap. J.*, **36**, 397–399, 1912.

[28] Clairaut's proof of this applied to the case when the distorting force arises from rotation and results in a zonal spherical harmonic of the second order; but, as any spherical harmonic of this order can be built up from zonal harmonics with different axes, the extension of the proof is obvious.

where g_0 is gravity at the distance r_0 from the center. To the first order,

$$\frac{r-b}{b} = u \cos^2\beta \sin^2\lambda - v \sin^2\beta\,, \tag{9}$$

where λ and β are longitude and latitude defined by the equations

$$x' = a \cos\beta \sin\lambda\,; \qquad y' = b \cos\beta \cos\lambda\,; \qquad z' = c \sin\beta\,,$$

where x', y', z' is a point on the surface.

In the integral for C, $\gamma d\sigma$ is the projection of the element $d\sigma$ on the plane $z' = 0$, which is $ab \sin\beta \cos\beta\, d\beta\, d\lambda$.

When $x = 0$ and $y = 1$, we then have

$$C = abJ_0 \int_0^{2\pi} d\lambda \int_0^{\pi/2} \left\{ 1 + \frac{2K-4}{2K+1} (u \cos^2\beta \sin^2\lambda - v \sin^2\beta) \right\} \sin\beta \cos\beta\, d\beta$$

$$= \pi abJ_0 \left\{ 1 + \frac{2K-4}{2K+1} (\tfrac{1}{4}u - \tfrac{1}{2}v) \right\}$$

$$= \pi b^2 J_0 \left\{ 1 + \frac{5K}{4K+2}\, u - \frac{K-2}{2K+1}\, v \right\}.$$

For projection on the plane $y' = 0$,

$$\gamma d\sigma = ac \cos^2\beta \cos\lambda\, d\beta\, d\lambda\,,$$

$$B = \pi acJ_0 \left\{ 1 + \frac{2K-4}{2K+1} (\tfrac{1}{4}u - \tfrac{1}{4}v) \right\}$$

$$= \pi b^2 J_0 \left\{ 1 + \frac{5K}{4K+2} (u - v) \right\},$$

and, similarly,

$$A = \pi bcJ_0 \left\{ 1 + \frac{2K-4}{2K+1} (\tfrac{1}{2}u - \tfrac{1}{4}v) \right\}$$

$$= \pi b^2 J_0 \left\{ \frac{K-2}{2K+1}\, u - \frac{5K}{4K+2}\, v \right\}.$$

We then have

$$\left. \begin{aligned} L_3 = \pi b^2 J_0 \Big\{ 1 &+ \frac{5K}{2+4K} (u - v) \\ &+ \frac{3K+4}{4K+2} (v \sin^2\delta - u \cos^2\delta \cos^2 t) \Big\}. \end{aligned} \right\} \tag{10}$$

If we set $x = 1$, $y = 1$, in (4), the integrands for A, B, C, must be multiplied by the appropriate values of $\cos \gamma$—that is, the direction-cosines l, m, n, of the normal. These are proportional to x/a^2, y/b^2, z/c^2. To the first order

$$\begin{aligned}
l &= \cos \beta \sin \lambda \ (1 - u + u \cos^2 \beta \sin^2 \lambda - v \sin^2 \beta) \,, \\
m &= \cos \beta \cos \lambda \ (1 \qquad + u \cos^2 \beta \sin^2 \lambda - v \sin^2 \beta) \,, \\
n &= \qquad \sin \beta \ (1 + v + u \cos^2 \beta \sin^2 \lambda - v \sin^2 \beta) \,.
\end{aligned}$$

We then find

$$\left.\begin{aligned}
L_4 = \tfrac{2}{3}\pi b^2 J_0 \Big\{ 1 &+ \frac{14K + 2}{10K + 5}(u - v) \\
&+ \frac{12K + 16}{10K + 5}(v \sin^2 \delta - u \cos^2 \delta \cos^2 t) \Big\} \,.
\end{aligned}\right\} \tag{11}$$

By (8) the gravitational effect should formally disappear if $K = 2$, and this substitution reduces (10) and (11) to (6) and (7).

We have, by (4),

$$\frac{J}{J_0} = (1 - x)(1 - y) + x(1 - y) \cdot \cos \gamma + y(1 - x) \cdot \frac{g}{g_0} + xy \cdot \cos \gamma \, \frac{g}{g_0} \,.$$

Hence, in the general case,

$$L = (1 - x)(1 - y)L_1 + x(1 - y)L_2 + y(1 - x)L_3 + xyL_4 \,,$$

or, substituting from (6)–(10) and neglecting the small constant term in u–v,

$$L = \pi b_1^2 J_0 \left\{ 1 - \tfrac{1}{3}x + (1 + \tfrac{1}{15}x)\left(1 + \frac{2 - K}{2 + 4K}\, y\right)(v \sin^2 \delta - u \cos^2 \delta \cos^2 t) \right\} ,$$

so that, in general, we may write

$$\left.L = L_0 \left\{ 1 + \left(\frac{1 + \frac{1}{15}x}{1 - \frac{1}{3}x}\right)\left(1 + \frac{2 - K}{2 + 4K}\, y\right)(v \sin^2 \delta - u \cos^2 \delta \cos^2 t) \right\} .\right\} \tag{12}$$

Here u and v are the actual ellipticities of the surface.

If our ellipsoid is a component of a binary, u and v are $(1 + 2K)$ times the values for a Roche ellipsoid, namely,

$$u_1' = \tfrac{3}{2}m_2 \frac{a_1^3}{m_1}\,; \qquad v_1' = \tfrac{1}{2}(m_1 + m_2)\frac{a_1^3}{m_1}\,. \tag{13}$$

We have also $\beta = (\pi/2) - i$, $l = \theta$ (the orbit longitude), so that

$$\left. \begin{aligned} L = L_0 \Big\{ 1 + \left(1 + \frac{6x}{15 - 5x}\right)&(1 + y + 2K - \tfrac{1}{2}yK) \\ &(v_1' \cos^2 i - u_1' \sin^2 i \cos^2 \theta) \Big\}\,. \end{aligned} \right\} \tag{14}$$

A similar equation holds good for the second component.

The combined light of the system (outside eclipse) may then be written

$$L = (L_1 + L_2)(1 - C \sin^2 i \cos^2 \theta)\,,$$

where we may set $L_1 + L_2 = 1$. Then, to the first order,

$$\left. \begin{aligned} C = \left(1 + \frac{6x_1}{15 - 5x_1}\right)&(1 + y_1 + 2K_1 - \tfrac{1}{2}y_1K_1)L_1u_1' \\ + \left(1 + \frac{6x_2}{15 - 5x_2}\right)&(1 + y_2 + 2K_2 - \tfrac{1}{2}y_2K_2)L_2u_2'\,. \end{aligned} \right\} \tag{15}$$

If the light between eclipses is expressed by the usual equation

$$L = a + b \cos \theta + c \cos^2 \theta\,,$$

we would have

$$\frac{c}{a} = C \sin^2 i\,, \tag{16}$$

were it not for the effect of "reflection" discussed by Eddington[29] and Milne.[30] Harmonic analysis of Milne's formula for the phase variation gives, for the combined light of the components viewed in the orbit plane,

$$\begin{aligned} L = 1 + (L_1a_2^2 - L_2a_1^2)&(0.347 \cos \theta + 0.005 \cos 3\theta) \\ &+ (L_1a_1^2 + L_2a_2^2)(0.073 \cos 2\theta + 0.002 \cos 4\theta)\,, \end{aligned}$$

[29] *M.N.*, **86**, 320, 1926.

[30] *M.N.*, **87**, 43, 1926.

while Eddington's formula substitutes ($0.333 \cos \theta + 0.000 \cos 3\theta$) and ($0.060 \cos 2\theta + 0.004 \cos 4\theta$).

The photometric ellipticity, corrected for reflection according to Milne, is then

$$C' = C + 0.15(L_1 a_2^2 + L_2 a_1^2) . \tag{17}$$

It is preferable to compute this term rather than to derive it from the value of b, as is often done, for two reasons: (1) when L_1 and L_2 are comparable, the reflection term in $\cos \theta$ becomes small, but that in $\cos^2 \theta$ does not; (2) there is often an observed asymmetry in the light-curve, represented by a term in $\sin \theta$, and we cannot be sure that the whole of the term $b \cos \theta$ arises from "reflection." The "Roche ellipticities" u_1' and u_2' may be calculated if the elements and masses are known. Equation (15) then gives us one relation with six unknown quantities.

When the components are nearly equal, it is probable that x, y, and k are nearly the same for both; but for a typical "Algol system," where the companion is larger, cooler, and less massive than the primary, it is probable that the darkening coefficient x differs for the two, and possible that the internal-constitution coefficient K differs, while nothing is known about the gravitational effect y.

Even if the parameters are the same for the two stars, we have one equation and three unknowns, and it is necessary to find two of these from other data.

From sufficiently good observations of the whole light-curve, the degree of darkening, x, may be found, though it is very hard to derive separate values for the two stars.

We have seen in Part I that K is small, while there is no doubt that x is often large (its theoretical value for total radiation being about 0.6), and y may well be greater. Hence, the photometric ellipticity appears to be quite unsuited for the determination of internal constitution. It should, however, in future, give a good determination of the gravity effect and so an independent datum for the testing of astrophysical theories.

The most recent discussion of the relation between dynamical and photometric ellipticity is Luyten's.[31] He has taken the gravity

[31] *M.N.*, **98**, 459, 1938.

effect into account by adopting a mean value of a correction factor but has allowed for darkening by the inadequate method of adopting the value of b/a given by each computer, without re-examining his assumptions. An exact comparison of his results with the present discussion is therefore impracticable.

Walter[32] has tabulated the photometric ellipticities on a uniform scale and has corrected them for the reflection effect. For uniform disks, and without gravity effect, he expresses the ratio of the photometric to the Roche-model result by a factor K_u, which is 3 for this and 7.5 for a homogeneous body, so that $K_u = 3(1 + 2K)$ on our scale. For the mean of 39 systems he finds $K_u = 6.52$, or $1 + 2K = 2.17$. On his assumptions this would indicate a low central condensation; but the present analysis gives the equation

$$\left(1 + \frac{6x}{15 - 5x}\right)(1 + y + 2K - \tfrac{1}{2}yK) = 2.17$$

(where the results will be some sort of average values for the two stars). On various assumptions this gives

$x =$ 0	1/2	1	0	1/2	1	0	1/2	1	0	1/2	1
$y =$ 0	0	0	1/2	1/2	1/2	1	1	1	1.17	0.75	0.36
$K =$ 0.58	0.37	0.18	0.38	0.14	−0.08	0.11	−0.17	−0.43	0	0	0

This shows the enormous range of uncertainty. If we set $x = 0.6$, we find

$$y = \frac{0.67 - 2K}{1 - 0.5K} .$$

For B and A stars the limb darkening should be comparable to that for the sun in the infrared, and hence small. As most eclipsing variables are of these types, the gravity effect may have nearly the theoretical value.

The gravity effect, if present alone, would make the center of the disk darker than the limb, at the time of eclipse, and so modify the light-curve.[33] From equations (3), (4), (8), and (9) it follows that, if we could see star 1 through star 2, the apparent surface brightness,

[32] *Schriften der Königsberger Gelehrten Gesellschaft*, **3**, 65, 1931.

[33] I owe this suggestion to a conversation with Dr. Kopal.

in the absence of the ordinary limb darkening, and neglecting v, would follow the law

$$J_g = J_1 \left\{ 1 + y(3K - 6) \frac{m_2}{m_1} a_1^3 \cos^2 \gamma \right\} .$$

For the reflection effect, at conjunction, Milne's formula[34] may be written in our notation

$$J_r = J_1 \left\{ 1 + \frac{1}{8} \frac{L_2}{L_1} a_1^2 (1 + 2 \cos \gamma)^2 \right\} .$$

The apparent laws of darkening are not the same—nor is either one of the usual form; but the ratio of the change from center to edge of the disk is

$$(3K - 6) \frac{L_1 m_2}{L_2 m_1} a_1 .$$

For equal stars and small k, the gravity effect will be somewhat the greater of the two, but the resultant of the two will give a limb brightening of the order of 0.1 at most, which would be masked by the ordinary darkening. The effect would be greater for the large faint component of a typical Algol system; but this would affect only the curve for the secondary minimum, which is too shallow to be determined with great percentage accuracy. It is doubtful, therefore, whether this effect is observable.

III. DIFFERENCES IN FORM OF THE COMPONENTS

It has been recognized for forty years[35] that, when the components of an eclipsing pair are ellipsoidal but differ in mass and density, they will differ also in form. To avoid complications in determining the elements, it is usually assumed that the components are similar in figure. The effects of this simplification have been discussed by Walter,[36] who concludes, from consideration of a particular case, that the effect on the light-curve is similar to that of darkening at the limb but is of opposite sign.

[34] *M.N.*, **87**, 50, 1926.

[35] G. W. Myers, *Ap. J.*, **7**, 8, 1898.

[36] *Schriften der Königsberger Gelehrten Gesellschaft*, **3**, 55–100, 1931 (especially pp. 82–89).

A fuller treatment, however, is desirable and turns out to be fairly simple, provided that it is assumed that the two ellipsoids present uniform disks and are prolate, with their major axes in the line of centers.

Let a_1 and a_2 be the major semi-axes of these ellipsoids, and b_1 and b_2 their minor semi-axes, a_1 and b_1 being the greater. The minor semi-axes of the projected ellipses will be b_1 and b_2 and the major d_1 and d_2, where

$$d_1^2 = a_1^2(1 - \epsilon_1^2 \sin^2 i \cos^2 \theta) ,$$

ϵ_1 being the eccentricity of the meridian section, i the orbital inclination, and θ the longitude from mid-eclipse. The projected distance of centers δ is given by

$$\delta^2 = \cos^2 i + \sin^2 i \sin^2 \theta , \tag{18}$$

whence

$$d_1^2 = b_1^2(1 + u\delta^2) , \tag{19}$$

where

$$u = \frac{\epsilon^2}{1 - \epsilon^2} , \tag{20}$$

and similarly for d_2.

Now let α be the fraction of the area of the smaller disk which is obscured by the larger (as for similar disks), and let

$$\delta = d_1 + p d_2 . \tag{21}$$

If $2s$ is the common chord and

$$\sin \varphi_1 = \frac{s}{b_1} ; \qquad \sin \varphi_2 = \frac{s}{b_2} , \tag{22}$$

the areas of the segments cut from the ellipses by this chord are

$$A_1 = b_1 d_1(\varphi_1 - \sin \varphi_1 \cos \varphi_1) ,$$

$$A_2 = b_2 d_2(\varphi_2 - \sin \varphi_2 \cos \varphi_2) ,$$

whence

$$\pi \alpha = \frac{A_1 + A_2}{\pi b_2 d_2} = \varphi_2 - \sin \varphi_2 \cos \varphi_2 + \frac{b_1 d_1}{b_2 d_2} (\varphi_2 - \sin \varphi_2 \cos \varphi_2) . \tag{23}$$

Also,

$$p = \cos \varphi_2 - \frac{d_1}{d_2} (1 - \cos \varphi_1) . \tag{24}$$

These equations are exact and, with (19) to (22), suffice for the precise computation of the light-curve in any given case.

We are concerned with the question of how closely such a curve can be represented by another due to the eclipse of two similar ellipsoids, by proper adjustment of the elements of the system. We may first find a pair 2 of similar ellipsoids such that the values of δ at the external and internal contacts are the same as for the original pair 1, calculate the light-curve for pair 2, and then adjust the elements so as to shift this curve into the best practicable agreement with the original. We will confine ourselves to terms of the first order in u_1, u_2.

Then

$$d_1 = b_1(1 + u_1\delta^2) . \tag{25}$$

Let b_1' and b_2' be the minor semi-axes of pair 2, and u' their common prolateness. Then, by (21),

$$\delta = b_1 + pb_2 + (u_1b_1 + pu_2b_2)\delta^2 , \tag{26}$$

$$\delta' = b_1' + p'b_2' + (u'b_1' + p'u'b_2')\delta'^2 . \tag{27}$$

If $\delta = \delta'$ at external contact ($p = p' = 1$), we have

$$b_1' - b_1 + b_2' - b_2 = \{b_1(u_1 - u') + b_2(u_2 - u')\}(b_1 + b_2)^2 .$$

At internal contact

$$b_1' - b_1 - (b_2' - b_2) = \{b_1(u_1 - u') - b_2(u_2 - u')\}(b_1 - b_2)^2 ,$$

whence

$$b_1' - b_1 = b_1 \{(b_1^2 + b_2^2)(u_1 - u') + 2b_2^2(u_2 - u')\} , \tag{28}$$

$$b_2' - b_2 = b_2\{2b_1^2(u_1 - u') + (b_1^2 + b_2^2)(u_2 - u')\} . \tag{29}$$

If, as usual in the theory of eclipsing variables, we set

$$k = \frac{b_2}{b_1}, \tag{30}$$

we have

$$k' - k = k(b_1^2 - b_2^2)(u_1 + u_2 - 2u') . \tag{31}$$

If, at any other time, $\delta = \delta'$, (26) and (27) give

$$p' - p = (1 - p^2)b_2\{b_1(u_1 - u') + (2b_1 - pb_2)(u_2 - u')\} . \tag{32}$$

Let L_1 be the light of star 1 of pair 1, when seen end-on ($\delta = 0$). Its light at any other time will be

$$\frac{L_1 d_1}{b_1} = L_1(1 + u_1\delta^2) ,$$

and that of the system outside eclipse will be

$$L_0 = L_1 + L_2 + (u_1 L_1 + u_2 L_2)\delta^2 .$$

As usual, we may assume $L_1 + L_2 = 1$.

For pair 2

$$L_0' = L_1' + L_2' + u'(L_1' + L_2')\delta^2 .$$

We will have $L_0' = L_0$ if $L_1' + L_2' = 1$, and

$$u' = L_1 u_1 + L_2 u_2 . \tag{33}$$

The figure to be attributed to pair 2 is thus defined by the "photometric ellipticity" derived from the uneclipsed part of the curve. We now have

$$u_1 - u' = L_2(u_1 - u_2) ; \qquad u_2 - u' = -L_1(u_1 - u_2) . \tag{34}$$

During the principal eclipse, of star 2 by star 1, the light will be

$$L = L_0' - \alpha L_2(1 + u_2\delta^2) . \tag{35}$$

To rectify the light-curve so that it is flat between minima, we must divide this by L_0 (or L_0'), obtaining

$$L_r = 1 - \alpha L_2\{1 + (u_2 - u')\delta^2\} .$$

The rectified loss of light during totality will be $L_2\{1 + (u_2 - u')\delta^2\}$, and not strictly constant. For pair 2 it will be simply L'_2.

If these are to agree at internal contact, we must have

$$L'_2 = L_2\{1 + (u_2 - u')(b_1 - b_2)^2\} ; \tag{36}$$

if, on the average, during totality

$$L'_2 = L_2\{1 + \tfrac{2}{3}(u_2 - u')(b_1 - b_2)^2\} .$$

Adopting the first assumption,

$$L_r = 1 - \alpha L'_2[1 + (u_2 - u')\{\delta^2 - (b_1 - b_2)^2\}] . \tag{37}$$

For the eclipse of pair 2 we have

$$L'_r = 1 - \alpha' L'_2 = 1 - L'_2 A(p', k') , \tag{38}$$

where the function A is defined by the standard equations

$$\left.\begin{aligned} p' &= \cos\varphi'_2 - \frac{1}{k'}(1 - \cos\varphi'_1) , \\ \pi A &= \varphi'_2 - \sin\varphi'_2\cos\varphi'_2 + \frac{1}{k^2}(\varphi'_1 - \sin\varphi'_1\cos\varphi'_1) , \\ \sin\varphi'_1 &= k'\sin\varphi'_2 . \end{aligned}\right\} \tag{39}$$

From (23) and (24) we have

$$p = \cos\varphi_2 - \frac{1}{k}(1 - \cos\varphi_1)\{1 + (u_1 - u_2)\delta^2\} ,$$

$$\pi\alpha = \varphi_2 - \sin\varphi_2\cos\varphi_2 + \frac{1}{k^2}(\varphi_1 - \sin\varphi_1\cos\varphi_1)\{1 + (u_1 - u_2)\delta^2\} ,$$

whence

$$\pi\alpha = \pi A\left(p + \frac{1}{k}(1 - \cos\varphi_1)(u_1 - u_2)\delta^2, k\right) \\ + \frac{1}{k^2}(\varphi_1 - \sin\varphi_1\cos\varphi_1)(u_1 - u_2)\delta^2 .$$

From (39) it follows that

$$\pi \frac{\partial A}{\partial p} = -\frac{2}{k} \sin \varphi_1 ; \qquad \pi \frac{\partial A}{\partial k} = -\frac{2}{k^3} (\varphi_1 - \sin \varphi_1) , \tag{40}$$

whence

$$\pi a = \pi A(p, k) + \frac{1}{k^2} (\varphi_1 - 2 \sin \varphi_1 + \sin \varphi_1 \cos \varphi_1)(u_1 - u_2)\delta^2 . \tag{41}$$

Also,

$$\pi a' = \pi A(p, k) - \frac{2}{k} \sin \varphi_1 (p' - p) - \frac{2}{k^3} (\varphi_1 - \sin \varphi_1)(k' - k) , \tag{42}$$

or, by (30)–(34),

$$\left.\begin{aligned}\pi a' = \pi A(p, k) + 2b_1^2(u_1 - u_2) \Big\{ & (1 - p^2) \sin \varphi_1 (L_2 - (2 + pk)L_1) \\ & - \frac{1 - k^2}{k^2}(\varphi_1 - \sin \varphi_1)(L_2 - L_1) \Big\} .\end{aligned}\right\} \tag{43}$$

Combining these results, we find

$$L_r - L_r' = L_2' b_1^2 (u_1 - u_2)(C_1 L_1 + C_2 L_2) , \tag{44}$$

where

$$\left.\begin{aligned}\pi C_1 = & -2(2 + pk) \sin \varphi_1 (1 - p^2) + \frac{2(1 - k^2)}{k^2} (\varphi_1 - \sin \varphi_1) \\ & - \frac{(1 + pk)^2}{k^2} (\varphi_1 - 2 \sin \varphi_1 + \sin \varphi_1 \cos \varphi_1) \\ & + \pi \{(1 + pk)^2 - (1 - k)^2\} a ,\end{aligned}\right\} \tag{45}$$

$$\left.\begin{aligned}\pi C_2 = & \; 2 \sin \varphi_1 (1 - p^2) - \frac{2(1 - k^2)}{k^2} (\varphi_1 - \sin \varphi_1) \\ & - \frac{(1 + pk)^2}{k^2} (\varphi_1 - 2 \sin \varphi_1 + \sin \varphi_1 \cos \varphi_1) .\end{aligned}\right\} \tag{46}$$

The elements of pair 2 which are subject to adjustment may be taken as b_2' and k', and the inclination as i'.

We have

$$\delta'^2 = \cos^2 i' \cos^2 \theta + \sin^2 \theta ,$$

where θ is an orbit longitude, which we must regard as an observed datum, not subject to change. We then have

$$1 - \delta'^2 = \cos^2\theta \sin^2 i' ; \qquad d\delta' = -\frac{1 - \delta'^2}{\delta'} \cot i' di' . \qquad (47)$$

If $l' = 1/k'$,

$$\delta' = b_2'(l' + p')(1 + u'\delta'^2 \ldots .) .$$

Hence, neglecting $u'd\delta'$, etc.,

$$d\delta' = (l' + p')db_2' + b_2'dp' + b_2'dl' , \qquad (48)$$

$$dp' = -dl' - (l' + p')\frac{db_2'}{b_2'} + \left\{ l' + p' - \frac{1}{b_2'^2(l' + p')} \right\} \cot i'di' .$$

Hence, by (40),

$$d\alpha' = \frac{\sin \varphi_1'}{l' + p'} X + \sin \varphi_1'(l' + p')Y + \varphi_1'Z , \qquad (49)$$

where

$$\left. \begin{aligned} X = \frac{2}{\pi k' b_2'^2} \cot i'di' ; \qquad Y = \frac{2}{\pi k'}\left(\frac{db_2'}{b_2'} - \cot i'di'\right) ; \\ Z = +\frac{2}{\pi k'} dl' , \end{aligned} \right\} \qquad (50)$$

or

$$\left. \begin{aligned} \cot i'di' = \tfrac{1}{2}\pi k' b_2'^2 X ; \qquad \frac{db_2'}{b_2'} = \tfrac{1}{2}\pi k'(Y + b_2'^2 X) ; \\ \frac{db_1'}{b_1'} = \tfrac{1}{2}\pi k'(Y + b_2'^2 X + k'Z) . \end{aligned} \right\} \qquad (51)$$

If x, y, and z are the coefficients of X, Y, and Z in (49), we have, for the rectified change in light of pair 2 at any time,

$$dL_r' = -L_2'd\alpha' .$$

Equating this to $L_r - L_r'$, we have

$$xX + yY + zZ = -b_1^2(u_1 - u_2)(C_1L_1 + C_2L_2) . \qquad (52)$$

These equations are best discussed numerically. Table 5 gives the relevant data for two typical ratios of radii: $k = 0.8$ and $k = 0.6$. By adjustment of X, Y, and Z the computed curve for the new pair 3 of similar ellipsoids can be made to cut the (hypothetically) observed curve for the dissimilar stars at any three desired points. These have been chosen to correspond to $\cos \varphi_2 = +0.6, 0.0, -0.6$. The results are as follows, the tabular unit being $b_1^2(u_1 - u_2)$:

	$k=0.8$		$k=0.6$	
	$L_1=1$	$L_2=1$	$L_1=1$	$L_2=1$
πX..........	-0.25	$+0.29$	-1.12	$+1.73$
πY..........	$+\ .18$	$-\ .21$	-0.14	$+0.45$
πZ..........	$+0.68$	-0.83	$+1.44$	-2.59

The residuals, in the sense O − C (pair 1 minus pair 3), are given in the table. For other values of L_1 and L_2 the results may be obtained by simple interpolation. When $L_1 = L_2$, X, Y, and Z and the residuals become very small. That is, the light-curve produced by eclipses of (prolate) ellipsoids of different sizes and shapes, but equal in luminosity, can be almost exactly represented by that of a pair of similar ellipsoids.

The greatest observable effect will occur when the smaller star is much the brighter (L_1 small). Even then, the residuals will be very small. To express them in terms of the light outside eclipse they must be multiplied by $1/\pi\{L_2'b_1^2(u_1 - u_2)\}$. In a "special Algol system" of this type it will be very rare to have $b_1 > 0.4$, or $u_1 > 0.3$, which makes the foregoing expression 0.024 at most. The largest residuals, near the beginning of eclipse, amount therefore to $0^m\!.006$ or less. Those close to totality, when expressed in magnitudes, may be larger; but it is to be remembered that the adjustment of the curves has been made in a summary fashion, and a complete adjustment, on least-square principles, would give a better fit.

For $k = 0.6$ the residuals are still smaller. The residuals indicate that the light of the dissimilar pair 1 near the beginning of the eclipse will be fainter than that of the similar pair 3. (It is brighter than

that of the intermediate pair 2, but the final adjustment reverses this in this region.)

TABLE 5

EFFECTS OF DISSIMILAR SHAPE

$\cos\varphi_2$	α	p	x	y	z	πC_1	Res.	πC_2	Res.	$D-U$
					$k=0.8$					
+1.00	0.000	+1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
+0.90	0.033	+0.821	0.17	0.72	.35	− .18	+ .15	+ .14	− .25	− .020
+0.80	0.091	+0.647	0.25	0.91	.50	.30	+ .14	.34	− .19	− .016
+0.60	0.249	+0.310	0.41	1.00	.70	.55	.00	.67	.00	.000
+0.40	0.413	0.000	0.59	0.92	.82	.68	− .10	.80	+ .10	+ .004
+0.20	0.580	−0.273	0.80	0.76	.90	.61	− .06	.74	+ .07	+ .002
0.00	0.723	−0.500	1.07	0.60	.93	.47	.00	.58	.00	− .001
−0.20	0.834	−0.673	1.36	0.45	.90	.30	+ .05	.39	− .05	− .002
−0.40	0.909	−0.800	1.63	0.33	.82	.16	+ .04	.23	− .04	.000
−0.60	0.965	−0.890	1.78	0.23	.70	.06	.00	.11	.00	− .001
−0.80	0.987	−0.952	1.61	0.14	.50	.02	− .05	.03	+ .06	− .001
−0.90	0.995	−0.978	1.28	0.10	.35	− .01	− .07	+ .01	+ .07	− .002
−1.00	1.000	−1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
					$k=0.6$					
+1.00	0.000	+1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000
+0.90	0.029	+0.849	.10	.68	.26	− .06	+ .12	+ .09	− .12	− .016
+0.80	0.081	+0.688	.15	.85	.37	− .16	+ .08	.22	− .09	− .010
+0.60	0.212	+0.395	.23	.99	.50	− .33	.00	.45	.00	.000
+0.40	0.361	+0.125	.31	.98	.58	− .41	− .04	.58	+ .04	+ .007
+0.20	0.508	−0.118	.38	.91	.63	− .40	− .03	.60	+ .03	+ .004
0.00	0.644	−0.333	.45	.80	.64	− .32	.00	.53	.00	− .002
−0.20	0.762	−0.518	.50	.66	.63	− .24	+ .02	.39	− .06	− .004
−0.40	0.856	−0.675	.55	.55	.58	− .13	+ .03	.27	− .03	− .001
−0.60	0.928	−0.805	.56	.41	.50	− .05	.00	.15	.00	+ .001
−0.80	0.977	−0.912	.48	.27	.37	.00	− .04	.05	+ .04	+ .003
−0.90	0.992	−0.951	.36	.19	.26	+ .01	− .02	+ .02	+ .05	+ .002
−1.00	1.000	−1.000	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.000

This also happens if we try to represent the eclipse of a disk darkened at the limb by one of a uniform disk. For the first we have

$$\sin^2\theta = A + B\psi(k, \alpha) .$$

If we denote the values for uniform disks by accents, we have also

$$\sin^2\theta' = A' + B'\psi'(k', \alpha') .$$

With the tables of the ψ-functions[37] it is easy to make the curves cut at any three desired points, and so find the difference $\alpha' - \alpha$ for any given value of α. The results are given in the last column of Table 5. The curves are made to cross at $\alpha = 0.25, 0.70$, and 0.95 for $k = 0.8$, and at $\alpha = 0.20, 0.60$, and 0.90 for $k = 0.6$—close enough to the intersections in the case of the dissimilar stars to give substantially the same type of adjustment. The difference of brightness, $D - U$, is here given in terms of L_2 as unit. The course of the curves of residuals is strikingly similar (as it has been practically forced to be). It appears that the residual effects of the difference of shape are very similar to those which would be produced by a slight darkening at the limb of amount approximately equal to twelve and eight times the unit of the residuals, which, in the extreme case just assumed, would amount to 0.3 or 0.2 in the coefficient of darkening.

When $u_1 - u_2$ is positive (as it is in the special Algol type), the effect simulates an increased limb darkening. This is opposite to Walter's conclusion; but he did not take into account the effects of rectifying the light-curve or those of adjustment of the elements—except to compare the effects of shape, without such adjustment, with those found by Dugan for limb darkening after adjustment.

By introducing such a degree of darkening into pair 3, its light-curve can be made to agree with pair 1 far within the accuracy of present observations (except perhaps when the minimum is very deep). To attempt to detect a difference of shape by photometric observations in a single wave length would therefore be almost hopeless, if we could be sure in advance what was the degree of darkening; otherwise it would be effectively impossible.

It does not seem necessary to compute additional cases—especially in view of the excellent agreement of these two.

For nearly equal radii the problem is more difficult. Equations (23) and (24) are no longer applicable if the radius of curvature of the larger disk, at the end of its major axis, is less than that of the smaller—that is, if $b_1/d_1^2 < b_2/d_2^2$. In this case, the last parts of the small disk to disappear are two symmetrical areas on each side of the axis. If this condition is nearly but not quite satisfied, the convergence of differential approximations will be bad.

[37] *Ap. J.*, **35**, 335, and **36**, 245, 1912.

When k' is small, we find (neglecting k'^4 in the parentheses)

$$x = k'^2 \sin \varphi_2' \{1 - k' \cos \varphi_2' + k'^2(1 - \tfrac{1}{2} \sin \varphi_2') - k'^3 \cos \varphi_2'\} ,$$

$$y = \sin \varphi_2' \{1 + k' \cos \varphi_2' - \tfrac{1}{2} k'^2 \sin^2 \varphi_2'\} ,$$

$$z = k' \sin \varphi_2' \{1 + \tfrac{1}{6} k'^2 \sin^2 \varphi_2'\} ,$$

$$\pi C_2 = k' \sin^3 \varphi_2' \{2 + \tfrac{8}{3} k' \cos \varphi_2' + k'^2(\tfrac{2}{3} - \tfrac{19}{15} \sin^2 \varphi_2')\} .$$

If

$$S = -\frac{x}{k'^3} - \frac{y}{k'} + \left(\frac{2}{k'^2} + 1 + k'^2\right) z = \tfrac{4}{3} k' \sin^3 \varphi_2' + k'^2 \sin \varphi_2' \cos \varphi_2' + k'^3(\tfrac{2}{3} \sin^3 \varphi_2' + \tfrac{2}{5} \sin^5 \varphi_2') ,$$

$$\pi C_2 - \tfrac{3}{2} S = \tfrac{8}{3} k'^2 \cos \varphi_2' \sin \varphi_2' (\sin^2 \varphi_2' - \tfrac{9}{16}) .$$

If

$$S' = -\frac{x}{k'} + k'y + k'^2 z ,$$

then, neglecting k'^4,

$$S' = 2k'^2 \cos \varphi_2' \sin \varphi_2',$$

whence

$$\pi C_2 - \tfrac{3}{2} S - mS' = \tfrac{8}{3} k'^2 \cos \varphi_2' \sin \varphi_2' (\sin^2 \varphi_2' - \tfrac{9}{16} - \tfrac{3}{4} m) .$$

We may thus make the computed curve intersect the given curve at any two points equidistant from the middle, as well as at the middle ($\cos \varphi_2 = 0$) and at the ends.

We will then have

$$\pi X = -\frac{3}{2k'^3} - \frac{m}{k'} ; \qquad \pi Y = -\frac{3}{2k'} + mk' ;$$

$$\pi Z = \frac{3}{k'^2} + \tfrac{3}{2} + (\tfrac{3}{2} + m) k'^2 ,$$

or, by (51), neglecting small terms,

$$\cot i' di' = -\frac{3}{4} \frac{b_2^2}{k'^2} = -\tfrac{3}{4} b_1'^2 ;$$

$$\frac{db_2'}{b_2'} = -\frac{3}{2k'} (1 + b_1'^2) ; \qquad \frac{db_1'}{b_1'} = \frac{3}{2k'} (1 - b_1'^2) .$$

These have to be multiplied by $b_1^2(u_1 - u_2)$. For very small values of k', they would nevertheless become large enough to vitiate the method of differential corrections, but this is of no practical importance.

A determination of the degree of darkening from photometric observations—even if they covered the whole light-curve and were of the exorbitant precision required—would give a value of the darkening coefficient increased by the effect described above.

Differences in shape may, however, be determined by simultaneous precise observations in two well-separated wave lengths, as Walter has recently shown.[38] If the degree of limb darkening and the coefficient of the gravity effect do not change with wave length, the problem is simple, for the individual photometric ellipticities of the stars will be constant, while the observed values are means weighted according to L_1, L_2. Such differences increase the number of unknowns and demand additional data. Very accurate observations of the principal minimum should determine the difference of the darkening coefficient for the bright component, irrespective of the shape of the companion, for it appears safe to assume that the geometrical aspects of the eclipse are independent of the wave length (except for stars with extended envelopes, like ζ Aurigae). The absolute darkening coefficient for the companion might be determined, in principle, from the secondary minimum, for the radius and luminosity of the smaller star may be found from a deep principal minimum alone. Sufficiently numerous and precise observations of a star like U Cephei or U Sagittae might, therefore, solve the whole problem—subject to the uncertainty aroused by the existence of the unexplained asymmetry displayed by some of the best-observed light-curves.

IV. SUMMARY AND CONCLUSIONS

1. In Part I it has been shown that in all short-period binaries for which apsidal motion has been detected or has been proved to be very slow, the effective internal concentration of density must be high. For the eight systems for which tolerably good values (not

[38] *Zs. f. Ap.*, **16**, 167, 1938.

marked with colons in Table 3) can be obtained the mean value of the constant K is 0.011. It follows from equation (3) that the superficial ellipticity of figure of such stars will be, on the average, only 2 per cent greater than that of the Roche ellipsoid (eq. [13]).

2. In Part II it is shown that the photometric ellipticity, derived in the usual way from the part of the light-curve outside eclipse, depends on the internal constitution (K), the degree of darkening at the limb (x), and the gravitational effect in surface brightness (y) (eq. [15]). The effect of internal constitution (if at all similar to that of the stars for which it can be determined) is wholly negligible in comparison with those of the others.

3. In Part III the effect of difference in shape of the two components of a close eclipsing pair are discussed, assuming them to be dissimilar prolate ellipsoids of different but uniform surface brightness. It is found that the light-curve can be represented within the errors of ordinary observations by the eclipses of two similar ellipsoids having an ellipticity which is the weighted mean of those of the actual components (eq. [33]). The small outstanding differences can be almost perfectly represented by a slight darkening at the limb.

It appears, therefore, that there is little or no hope of determining precise geometrical figures of the components of an eclipsing pair from photometric observations. Greater accuracy can undoubtedly be obtained (when the masses are known) by calculating the geometrical ellipticity by equation (3) (assuming $K = 0.01$ if the refinement appears worth while) and utilizing the photometric observations for the study of darkening at the limb and of the gravity effect.

In the determination of orbital elements for eclipsing pairs the light-curve is usually rectified with the photometric ellipticity constant z derived from the non-eclipse portion, and the same constant is introduced into the geometrical equations for the determination of the radii and inclination. It would probably be a better approximation to use the value of z computed dynamically (with $K = 0.01$) in these equations. The results will not be rigorous in any case, for in the discussion of eclipses of ellipsoidal stars the assumption has been made, to avoid great complications, that the contours of equal

apparent brightness on the disks are ellipses similar to, and concentric with, the limb. An exact calculation, taking into account the effects of gravity as well as darkening, would be very complicated.

This problem has been discussed by Takeda[39] in considerable detail. His work deserves careful consideration by anyone who pursues the question. Very accurate observations—preferably in widely separated wave lengths—will probably be required if definite results are to be reached.

PRINCETON UNIVERSITY OBSERVATORY
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[39] *Mem. College Sci., Kyoto Imperial U.*, A, **20**, 47, 1937.

THE DISSIPATION OF PLANETARY FILAMENTS

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ABSTRACT

An investigation is made of the forces acting on a cylindrical filament of stellar matter produced by an encounter between two stars. It is shown that under rather general assumptions such a filament will expand under its own internal pressure much more rapidly than it will lose energy either by radiation or by turbulent convection. A similar conclusion may be deduced for a thin ribbon-like filament produced by a grazing collision. It seems probable that a stellar encounter will simply produce an extended gaseous nebula around one or more of the stars involved.

Since Lyttleton's[2] important paper there has been a renewed interest in encounter theories of the origin of the solar system. The papers by Luyten[3] and by others which have followed have dealt primarily with the stellar dynamics of the proposed encounter and have been largely concerned with difficulties of energy and momentum.

It is also relevant to investigate the conditions under which a filament of stellar matter, presumably torn out of both stars by the encounter, can condense into planets. Early work[4] on this phase of the problem involved the assumption that the temperatures of the filaments were of the same order as those of the solar photosphere. As Russell[5] has stressed, however, this assumption is certainly incorrect, since a filament with as much mass as Jupiter must come from layers in which the stellar temperature is at least 10^6 degrees. This fact had also been discussed by Jeffreys,[6] who showed that the planets would expand to radii comparable with the distances of their satellites. Jeffreys discussed only equilibrium configurations, however, and did not investigate either the rate of expansion or the rate of loss of internal energy by radiation.

[1] This work was done while the author was holding a National Research Fellowship.

[2] *M.N.*, **96**, 559, 1936.

[3] W. J. Luyten and E. L. Hill, *Ap. J.*, **86**, 470, 1937; *M.N.*, **99**, 692, 1939; R. A. Lyttleton, *M.N.*, **98**, 536, 1938.

[4] H. Jeffreys, *The Earth*, p. 27, Cambridge University Press, 1929.

[5] *The Solar System and Its Origin*, p. 112, New York: Macmillan, 1935.

[6] *M.N.*, **89**, 731, 1929.

As will be shown hereinafter, the gases within a filament at a temperature of hundreds of thousands of degrees will accelerate outward and within a few hours will reach the velocity of escape. The filament will then presumably dissipate. This fate might perhaps be avoided by the radiation of energy at a sufficiently rapid rate. The optical depth of any filament with appreciable mass, however, will be at least several millions, and the time required to radiate half the internal energy of the filament would be at least several months if there were no expansion. Since no convection currents with velocities less than the escape velocity can dispose of so much energy in a few hours, the filament must therefore explode.[7]

In section 1 of this paper it is shown that a filament produced by an encounter has, in general, a positive energy, i.e., more than enough to dissipate the entire filament to infinity. Section 2 presents the formulae for the rate of expansion of a cylindrical filament of circular cross-section on the assumption that no radiative loss of energy occurs. In section 3 the possibility of convection or turbulence is considered, and a solution of the equation of radiative transfer is given for the case in which the intensity of radiation is a function of the time; it is shown that the rate of energy loss is less than a hundredth of that required to avert the dissipation of the filament. Section 4 extends the analysis to a ribbon-like filament. Section 5 discusses the relevance of these results to the encounter theory of the origin of the solar system.

1. It is evident that, if a gaseous filament has a positive energy, the filament will expand to infinity (provided γ exceeds $4/3$) unless

[7] This result is reminiscent of A. W. Bickerton's theory of partial impact (*Trans. N. Zealand Inst.*, **11**, 125, 1878), in which the appearance of a nova was attributed to the explosion of such a filament. The origin of the planets, however, was also attributed to this process. Bickerton deserves mention for what is probably the first detailed statement of the encounter theory of the formation of the solar system.

The expansion of such a filament has also been discussed by F. Nölke in his thorough investigation of encounter theories (*Der Entwicklungsgang unseres Planetensystems*, pp. 188–191, 1930). His analysis is primarily intended, however, to show that the filament will leave the gravitational field of the sun, and he considers the motion of a point mass in the gravitational field of the two stars, neglecting both the mass and the internal pressure of the filament. Nölke's investigation is thus concerned with the same dynamical difficulties that Lyttleton and Luyten have more recently discussed, and has no direct connection with the present analysis.

it loses energy at a sufficiently rapid rate. If a filament is to be unstable, then, it is a necessary condition that the ratio of the internal positive energy to the absolute value of the negative gravitational energy exceed unity. A lower limit for this quantity is easily calculated.

One may assume that the matter in the filament, assumed to be of total mass M, was originally in one of the two stars involved in the supposed encounter, and that this matter had originally a maximum density ρ_{st} and a maximum temperature T_{st}. Since, in general, the temperature within a star decreases outward much less rapidly than the density, the mean stellar temperature of the matter in the filament may be taken as not less than $\frac{1}{2}T_{st}$. (In the standard model, for instance, the mean temperature for the entire star is 0.585 times the central temperature.)

The total internal energy W of the filament will then be not less than $3kM(\frac{1}{2}T_{st})/2\mu m_0$ ergs, where μ is the mean molecular weight, m_0 is the mass of unit atomic weight, and k is the usual Boltzmann constant. If γ, the ratio of the specific heats, is less than $5/3$, W is, of course, considerably greater than the foregoing lower limit.

The self-gravitational energy $-\Phi$ of the matter in the filament will have an absolute value not greater than that of a sphere of uniform density ρ_{st} with a radius S equal to $(3M/4\pi\rho_{st})^{1/3}$, for which the potential energy is $-3GM^2/5S$, where G is the gravitational constant. If we use the well-known formula (I.C.S. 87-1) for T^3/ρ in a gaseous star, assume that radiation pressure is negligible, and use the Eddington mass-luminosity relationship (I.C.S. 84-6), a minimum value of W/Φ is given by

$$W/\Phi > 0.283\left(\frac{M_0}{M}\right)^{2/3}, \qquad (1)$$

where M_0 is the mass of the parent-star. This is greater than unity when M is less than $0.15M_0$.

We see that W/Φ is greater than unity for small masses. If M is comparable with M_0, a separate investigation is required. Let us consider the most favorable case, in which a star is split by an encounter into two spheres of equal mass. From the virial theorem it

follows that, if radiation pressure is negligible, W/Φ for the parent-star is 0.50—almost twice as large as the lower limit found above when M is set equal to M_0. After the encounter the original kinetic energy will be divided equally between the two spheres. The potential energy of each, however, will be $(\frac{1}{2})^{5/3}$ times the former value of $-\Phi$, and hence W/Φ for each sphere will be 0.79. Unless the stellar matter consists entirely of hydrogen and helium, the additional energy of ionization and excitation would suffice to raise W/Φ well above unity.

In addition, the encounter itself may be expected to increase W/Φ even further. If the passing star is sufficiently massive and energetic to split the original star into two equal masses, it will presumably give gravitational energy to each component, decreasing Φ. Physically this means that, if a star is torn in two by tidal action, one may expect each component to be enormously elongated and hence to have much less gravitational energy. The internal energy W could not be decreased by the encounter, since superelastic collisions seem impossible.

In other words, a gaseous star A in equilibrium has a reserve of internal energy equal in magnitude to at least one-half its gravitational energy. A catastrophic encounter with star B will divert energy to A from the mutual motion of A and B. This will result in an enormous decrease of the gravitational energy of A, reflected partly in the disruption of A and partly in the elongation of each component of A. If, moreover, the encounter is a grazing one, the thermal energy of A will be increased. As a result, the internal energy will exceed the value of the negative gravitational energy, and either or both components will explode.

One may conclude, therefore, that any mass of stellar matter torn loose from one star by the passage of another will have more than enough energy to dissipate itself to infinity. Radiation of energy, however, will decrease the value of W; whether or not a filament formed by an encounter would actually disrupt completely therefore depends on the rates both of expansion and of radiation.

2. In order to calculate the rate of expansion, certain assumptions of uniformity are apparently necessary; these simplifications should

not alter the order of magnitude of the results. One may consider an idealized cylindrical filament of circular cross-section stretched between two stars, one of which is receding with a velocity V relative to the other. Let M, R, and z denote the mass, radius, and total length of the filament; M', R', and z' will be used to denote the same quantities in units of the solar mass and radius. The masses and radii of the two stars are left unspecified, as they play no primary part in the expansion of the filament.

The material in the filament will be acted upon by the pressure of the gases in the filament and by the gravitational attraction both of the filament itself and of the two stars. Only those forces will be considered which act in the radial direction in the filament, perpendicularly to the axis of the filament. The equation of motion in this simplified case is

$$a = -\frac{1}{\rho}\frac{\partial p}{\partial r} - \frac{\partial \varphi}{\partial r}, \tag{2}$$

where r is the perpendicular distance from the axis of the filament, a is the radial acceleration, p the pressure, and φ the gravitational potential. If we assume that all quantities are functions of r and t alone, the mass average of the acceleration, which will be denoted by $\bar{a}$, becomes

$$\bar{a} = -\frac{2\pi z}{M}\int_0^R \left(\frac{\partial p}{\partial r} + \rho\,\frac{\partial \varphi}{\partial r}\right) r dr\,. \tag{3}$$

The integral of the first term on the right-hand side of (3) will be denoted by $\bar{a}_1$, that of the second, by $\bar{a}_2$. If now we integrate by parts for $\bar{a}_1$ and substitute $\pi R^2 z\rho_m$ for M, where ρ_m is the mean density, we have

$$\bar{a}_1 = \frac{2}{R^2\rho_m}\int_0^R P dr = \frac{2}{R\rho_m}\,\overline{P}\,. \tag{4}$$

P_m, the average of P over the volume, will be less than $\overline{P}$ as defined in (4), since the large values of P for small r are weighted much more heavily in $\overline{P}$ than in P_m. We may therefore find a lower limit for $\bar{a}_1$ by setting $\overline{P}$ equal to P_m. As shown in Figure 1, T does not vary ap-

preciably throughout most of the filament, and we may legitimately assume that T is constant. With these two assumptions, we have

$$\bar{a}_1 = \frac{2kT}{\mu m_0 R}, \tag{5}$$

where the symbols have the same meanings as before. If the filament has been assumed to expand adiabatically, we know that

$$T = T_{st}\left(\frac{\rho}{\rho_{st}}\right)^{\gamma-1}, \tag{6}$$

where γ is the ratio of the specific heats. Formula (6) is rigorous only for the portion of the filament which was initially at the maximum temperature T_{st} and had the initial maximum density ρ_{st}. To an adequate approximation, however, this formula may be applied to the mean temperature and density of the gases in the filament.

To evaluate $\bar{a}_2$ we appeal to the virial theorem. This theorem states that a gaseous system expands when its thermal kinetic energy W is greater than $\frac{1}{2}\Phi$, where $-\Phi$ is again the gravitational energy. Since in the previous section it was shown that W is actually greater than Φ, one may deduce that $\bar{a}_1$ is at least twice as great as that part of $\bar{a}_2$ which arises from the gravitational attraction of the filament on itself. It is therefore possible to neglect this gravitational attraction in the computation of the rate of expansion; the rate so calculated will be too great by a factor of less than 2.

The gravitational attraction which the two stars exert on the filament is difficult to discuss in detail but is not likely, in any case, to prevent the expansion. Only the radial component of this attraction is effective in this connection, and since the force perpendicular to the line connecting the two stars varies as the inverse cube of z, it is evident that as soon as z' is much greater than unity such a force will be negligible.

We may let t_0 be the time at which this stellar gravitational field becomes negligible and the filament begins to expand. Similarly, ρ_0 and z_0 will be used to denote values of ρ_m and z at the time t_0. If

t_1 denotes the value of t for which the radial velocity outward, v, equals the velocity of escape from the filament, v_∞, then we have

$$v_\infty = \int_{t_0}^{t_1} a(t) dt . \tag{7}$$

If we neglect $\bar{a}_2$, assume that a_1 is constant through time, and replace v_∞ by its value for a spherical mass M of radius R, we find from (5), (6), and (7) that

$$t_1^* - t_0 = \left(\frac{\rho_{st}}{\rho_0}\right)^{\gamma-1} \frac{\mu m_0}{k T_{st}} \left(\frac{MGR}{2}\right)^{1/2} . \tag{8a}$$

Substituting $M/\pi R^2 z_0$ for ρ_0, we have

$$t_1^* - t_0 = 1.70 \cdot 10^{10} (\rho_{st} z_0')^{2/3} R'^{11/6} M'^{-1/6} T_{st}^{-1} \text{ sec} , \tag{8b}$$

where R', z', and M' again denote $R/R_\odot$, $z/R_\odot$, and $M/M_\odot$, respectively; t_1^* denotes the value of t_1 derived on the assumption that ρ is constant with time; and γ in (8b) has been set equal to 5/3, its value for a monatomic gas. We assume throughout that μ is unity. If T_{st} is 10^6 degrees, M' is 1/500, ρ_{st} is 1/10, and other quantities are of order unity, $t_1^* - t_0$ will equal approximately three hours. Since v_∞ is of the order of 30 km/sec, the change in R during this time will be $\frac{1}{4} R_\odot$; the change in z during the same period will, of course, be considerably larger.

More realistically, we may assume that ρ is proportional to $1/z$ and that z increases uniformly at a rate V, the velocity of recession of the one star relative to the other when t is equal to t_0. The time origin is chosen so that z equals Vt. In this case t_0 must be at least as great as the value of t for which the filament ceases to acquire additional mass, the filament expanding along its axis as the stars recede. With this assumption we obtain from (7)

$$t_1 = t_0 \left\{ 1 + (2 - \gamma) \frac{t_1^* - t_0}{t_0} \right\}^{1/(2-\gamma)} , \tag{9}$$

where $t_1^* - t_0$ is given in (8) above. The quantity t_0 is equal to z_0/V; if V is 10^8 cm/sec and z_0 is $2R_\odot$, t_0 equals $1.4 \cdot 10^3$ sec, or about half an hour.

3. One may consider next the loss of energy from the filament. It will be shown that neither convection currents nor radiative transfer can be responsible for an appreciable loss of energy from the filament in the short space of a few hours.

Convection currents are easily disposed of. No current of matter can reach the surface of the filament from the far interior before the time t_1^* unless the velocity of the current is greater than the velocity of escape from the filament. More quantitatively, if the condition that $(t_1^* - t_0)v_\infty$ be less than R is combined with (7) and with the formula for v_∞ in the spherical case, this gives roughly the condition that $\bar{a}_1$ be greater than $2\bar{a}_2$, which, as we have already seen, is fulfilled in all relevant cases. Hence, if such currents are responsible for any important transfer of heat, they must leave the filament entirely and will promote the dissipation of the filament rather than hinder it.

A number of approximations reduce the problem of radiative transfer to a tractable form in the present case. As a first approximation we assume that the filament is static. Then the equation of radiative transfer, combined with the equation of radiative equilibrium, gives

$$\nabla \cdot (\sigma \nabla J) = \frac{\partial}{\partial t}\left(\frac{J}{\xi}\right), \tag{10}$$

where

$$\sigma = \frac{c}{3\rho k}; \tag{11}$$

J is the integrated intensity of radiation, a function of position and of time; k is the Rosseland mean absorption coefficient; ξ is the ratio of the energy density of radiation to the total energy density within the filament; and c and ρ are the velocity of light and the density of matter, respectively.

In the present case it will be assumed that σ and ξ are constant throughout the filament, which is again assumed to be cylindrical and of circular cross-section; J is taken to be a function only of r, the distance from the filament axis. With these simplifications, (10) becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial J}{\partial r}\right) = \frac{1}{\xi\sigma}\frac{\partial J}{\partial t}. \tag{12}$$

A solution of (12) is given by

$$J(r, t) = Ae^{-\lambda t} J_0 \left(\frac{\lambda^{1/2} r}{\xi^{1/2} \sigma^{1/2}} \right), \tag{13}$$

where $J_0(x)$ is the Bessel function of the first kind of zero order, and A is an arbitrary constant. From the usual boundary condition that J equals $2H$, or $-2\sigma/c \cdot \partial J/\partial r$, when r equals R, we find that λ is determined by the equation

$$\lambda = \frac{cy^2\xi}{3\tau R}, \tag{14}$$

where τ is the optical depth ρkR from the surface of the filament to the center and where y is the solution of the equation

$$y = \frac{3\tau}{2} \frac{J_0(y)}{J_1(y)}. \tag{15}$$

When τ is very large, y is practically equal to any of the zeros of $J_0(y)$.

For the lowest mode of radiative decay y approaches 2.405 as τ approaches infinity. This gives the mode of longest half-life even

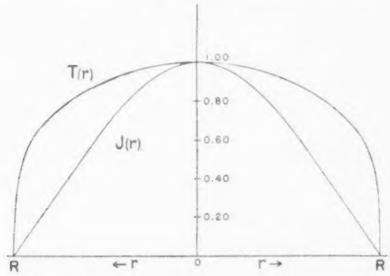

FIG. 1.—The upper and lower curves represent $T(r)$ and $J(r)$, respectively, taken along the diameter of the filament. The boundary temperature is assumed to be negligibly small.

when the variation of J with the other co-ordinates of the filament is taken into account. Values of $J(r)$ and of $T(r)$, or $J^{1/4}(r)$, for this lowest mode are shown in Figure 1 for a cross-section of the filament.

One may define t_2 as the time at which the energy density equals one-half the value it had at the time t_0, where t_0 is again the time at which the filament started to expand; zero subscripts will again be

used to denote quantities at the time t_0. Only the lowest mode need be considered, since the others decay more rapidly. If we combine (14) with the condition that $\lambda(t_2 - t_0)$ equals ln 2, or 0.693, then we find

$$t_2^* - t_0 = \frac{2.08R^2\rho_0 k}{cy^2\xi}, \tag{16a}$$

$$= \frac{1.09 \cdot 10^{11} M' k}{z_0' \xi} \text{ sec }, \tag{16b}$$

where the asterisk again denotes a value derived on the assumption that ρ is constant. Since k cannot be less than 0.19 (its value for electron scattering) and M' is at least 10^{-3}, $t_2^* - t_0$ is scarcely less than a year.

If the gases in the filament are to condense into planets, they must radiate more than half their internal energy before the time t_1, and the ratio of $t_2 - t_0$ to $t_1 - t_0$ must be considerably less than unity. When ρ and k are assumed constant through time, this ratio is given by (8b) and (16b), which yield

$$\frac{t_2^* - t_0}{t_1^* - t_0} = \frac{6.42 M'^{7/6} k T_{st}}{z_0'^{5/3} R'^{11/6} \rho_{st}^{2/3} \xi}. \tag{17}$$

Since k should be at least unity, k/ξ may be given a minimum value of 10. We may equate M to one-half the solar mass exterior to the radius at which the density and temperature before the encounter were ρ_{st} and T_{st}, and determine these quantities from the usual standard model. If z' and R' are assumed to be less than 10 and unity, respectively, we find that $(t_2^* - t_0)/(t_1^* - t_0)$ is at least 80,000.

If a uniform longitudinal expansion of the filament is also considered, J', or J/ρ, must be substituted for J in (10) and an extra term, which for small ξ reduces to $(\gamma - 1)\ J'/\xi t$ must be added to the right-hand side to take account of the work done per unit time per unit mass in the course of the expansion. The absorption coefficient k will also vary in this case but will, in general, increase during an adiabatic expansion, provided that some formula of the form $\kappa\rho/T^{7/2}$ is approximately valid for k. Hence, to find a lower limit for $t_2 - t_1$, we may assume k to be constant. If t_2 is defined as the time at

which the total internal energy is one-half of what it would have been had there been no radiation emitted since the time t_0, the analysis leads approximately to formula (9), with t_2 replacing t_1, t_2^* replacing t_1^*, and δ replacing γ, where δ equals $3\gamma - 4$.

To find an extreme minimum value for $(t_2 - t_0)/(t_1 - t_0)$—a value which, in fact, is less than any actually possible—we may use (9) in both cases, taking an upper limit for t_1 and a lower limit for t_2. These limits are reached for $\gamma = 5/3$ and $\delta = 0$ (corresponding to $\gamma = 4/3$), respectively. If t_1^* and t_2^* are both considerably greater than t_0, this lower limit becomes

$$\frac{t_2 - t_0}{t_1 - t_0} = 38.2 t_0^{5/2} \frac{(t_2^* - t_0)^{1/2}}{(t_1^* - t_0)^3}, \tag{18a}$$

$$= \frac{3.27 \cdot 10^3 M' k^{1/2} T_{\rm st}^3}{\xi^{1/2} \rho_{\rm st}^2 R'^{11/2} V^{5/2}}. \tag{18b}$$

Even if V is as great as 10^8 cm/sec, formula (18b) still gives a value greater than a hundred, provided values of M, $\rho_{\rm st}$, and $T_{\rm st}$ are again determined from the standard polytropic model for the sun, while R' and k/ξ are taken to be not greater than unity and not less than 10, respectively. Thus, we see that in general the idealized filament discussed here is unable to radiate an appreciable fraction of its energy in a sufficiently short time to avert complete dissipation.

4. The preceding arguments have been developed for the case of filaments with circular cross-section. It has been suggested[6] also that a very close encounter or grazing collision might produce a thin ribbon or sheet of stellar matter connecting the two stars. Essentially the same analysis may be applied in this case as well.

There is no need, however, to derive another set of formulae applicable to this case. It is evident from (18b) that for a filament of given mass M and given length z, the least value of $(t_2 - t_0)/(t_1 - t_0)$ is attained for maximum radius R. It also follows from general principles that the value of this ratio will increase as the initial cross-section of the filament is deformed from a circle of radius R to an ellipse with semi-major axis R and minor axis h, keeping the mass of the filament constant. In the first place, it is clear from dimensional

arguments that (14) must be a general formula in which y is a numerical constant of the order of unity, dependent on the geometry of the particular case envisaged, and in which $2R$ is the least dimension of the radiating system, which in this case is h. (For a slab of thickness $2R$ it may be shown that y is equal to $\pi/2$.) The optical depth τ will remain constant during this deformation, provided k is assumed to be constant; λ will vary roughly as $1/h$; and $t_2^* - t_0$ will decrease proportionally to h.

While the escape velocity will not be greatly affected by this transformation, the acceleration will be considerably changed. The pressure will increase as ρ increases; the accompanying increase in T will heighten this increase of P. The acceleration is proportional to the increase of pressure inward per unit element of mass, and in the course of the deformation it will change proportionally to the pressure itself. Since $t_1^* - t_0$ will therefore decrease somewhat more rapidly than h, the ratio $(t_2 - t_0)/(t_1 - t_0)$ will increase slightly as h is decreased. Hence, if this ratio is greater than 100 for any relevant circular cylindrical filament of diameter $2R$, it will be even greater than this lower limit for any ribbon of width equal to $2R$.

Convection may also be ruled out in this case. The acceleration, as we have just seen, increases roughly as $1/h$. Since the escape velocity is relatively unchanged by the deformation, the time $t_1 - t_0$ during which the convection currents must be effective, if they are to prevent the complete dissipation of the filament, decreases accordingly as h. Since this decrease of $t_1 - t_0$ balances out the decrease in h, the argument of section 3 may again be applied; i.e., if the currents convey enough matter to the surface to produce an appreciable effect, their velocity must be greater than the velocity of escape from the filament. From these considerations and from the conclusions reached in section 3, one may infer that a cylindrical filament will be, in general, unable to avert expansion and dissipation, regardless of what its cross-section may be.

It is of interest to note the magnitude of these effects. For a filament of length $R_\odot$, of width $2R_\odot$, of thickness $R_\odot/50$, of mass $M_\odot/500$, and for which $\rho_{st}^{2/3}/T_{st}$ equals 10^{-6}, the acceleration is so great that less than two minutes are sufficient for the gases to acquire the escape velocity of roughly 35 km/sec; during this time the thick-

ness of the filament increases, on the average, not more than 15 per cent.

5. The foregoing analyses are based on several approximations, but there is a considerable margin of safety in the conclusions. Even under extreme conditions there is a difference of two orders of magnitude between the short time required to produce macroscopic velocities sufficient to disrupt the filament and the long interval necessary to radiate half the internal energy. It is not likely that any or all of the various approximations made could introduce so large a factor.

It is difficult, therefore, to see how the filament as a whole can avoid dissipation. Although only the average acceleration has been discussed, it is unlikely that any part of the filament can remain after most of it has been forced off into space. The radial acceleration near the center of the filament is small, to be sure, but the velocity of escape decreases linearly with r for a uniform filament. Furthermore, the central gases will remain at the highest temperature for the longest time and will thus have more chance for expansion.

Nor is it probable that the outer portions of the filament might condense into planets as the filament expanded. Any sphere of gas in the expanding filament will itself be expanding in a direction perpendicular to the radius vector of the filament with a velocity directly proportional to the radius of the sphere. But the velocity of escape from any sphere will also vary directly as the radius. Thus, all spheres within the filament have the same chance of condensing, independently of their radii. The filament as a whole, however, and hence a sphere with a radius equal to that of the filament, cannot condense after the time t_1 has been reached. It follows that, as soon as the gases of the filament have reached the velocity of escape from the filament, no large portion of the filament can condense, independently of the temperature to which it may fall.

If, then, such a filament would not be likely to condense into planets, what would be the fate of the matter so ejected? Such of it as did not fall back into the stars or dissipate into free space would form an enormously extended atmosphere of some sort around one of the two stars directly involved in the encounter or around their possible companions. With an assumed mass of $2 \cdot 10^{30}$ grams and a

radius of 30 astronomical units, such an atmosphere would have a mean density of $5.4 \cdot 10^{-15}$ gm/cm^3 and would be comparable with the solar chromosphere. Such an atmosphere is reminiscent of the Laplace nebular hypothesis, except that in this case there need be no lack of angular momentum. The validity of the encounter theory as an explanation of the origin of the solar system rests apparently on whether or not a non-uniformly rotating atmosphere could condense into solid bodies.

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PROPER MOTIONS IN THE GALACTIC CLUSTER NGC 2548

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ABSTRACT

Proper motions are derived on Yerkes plates for an area 30′ in diameter centered on the cluster NGC 2548. The results are contained in Table 2. Plots of the motions are found in Figure 1, *A* and *B*. The positions of the probable cluster members are found in Figure 2, *A* and *B*. Figure 3 is the spectrum-magnitude diagram of the cluster. The luminosity function of the cluster is given in Figure 4.

This paper gives the results of the determination of proper motions in the cluster NGC 2548 ($\alpha = 8^h08^m.8$, $\delta = -5^\circ30'$, 1900; $l = 196^\circ$, $b = +17^\circ$). For this cluster Dr. R. J. Trumpler gives a distance of 470 parsecs, a diameter of 30′, and the type is designated as 1–2*a*, indicating that a small number of giant stars may belong to the cluster. Verification of the membership of these giants is one of the main objects of this paper.

I. THE MATERIAL

The material available for measurement consisted of four pairs of plates, taken with the 40-inch refractor of the Yerkes Observatory, each pair having only one image. Table 1 gives a list of these plates.

TABLE 1

THE MATERIAL

Old Plate	New Plate	Interval in Years	Limiting Photovisual Magnitude	Symbol
F 120	F 500	27.9	12.5	P I
F 176	F 499	24.9	12.0	P II
II 3048	F 498	21.0	11.6	P III
II 3095	F 595	21.8	12.0	P IV

II. MEASUREMENT AND REDUCTION

The procedure adopted in this paper is the same as that used in the measurement and reduction of the material for NGC 752.[1] Brief-

[1] *Ap. J.*, **89**, 431, 1939.

ly, that procedure is as follows: Each old plate was matched by a new plate taken through the glass. The two plates were placed film to film, and the measurements of Δx and Δy were made in both the direct and the reversed directions. The measured values of the differences were then averaged for each star, to form a mean value for both Δx and Δy. The measuring device was the same as that used for NGC 752. The reduction constants were obtained by a least-squares solution of the formula

$$\overline{\Delta x} = ax + by + c \;,$$

with a similar formula for the y-co-ordinate.

In order to select the stars of small motion for the least-squares solution, a preliminary reduction was first made for P I, the plate with the faintest limiting magnitude. Of the 114 stars on this plate, 76 were selected as comparison stars; for the remainder of the pairs—P II, P III, and P IV—the comparison stars were always taken from this group. The smallest number of comparison stars used was 38 for P III.

Then for each plate and each star the computed values of the differences (Δx_c and Δy_c) were obtained, and the proper motion in both co-ordinates (μ_x and μ_y) resulted from the difference between the observed and the computed value of each difference. With the aid of the scale value (1 mm = $10\overset{''}{.}6$) and the interval in years, the values of μ_x and μ_y were converted into units of ten-thousandths of a second of arc per year.

In order to select cluster stars for the elimination of the magnitude equation, the measures of all stars common to all four plates were averaged and plotted. The resulting plot indicated a very distinct concentration of stars near the origin, and from these stars 41 were selected as provisional cluster stars. The plate and filter combination used for the plates of this cluster give magnitudes very nearly on the photovisual system; but since no photovisual magnitudes were available, the measured diameters were used in the reductions. Subsequently, these diameters were converted into approximate photovisual magnitudes by means of the A-type stars

and their photographic magnitudes. For only two plates, P I μ_δ and P III μ_δ, was any magnitude equation detectable.

After the magnitude equation had been removed, the material was ready for the determination of the weights of the individual plates. Again the same plan was followed as in the case of NGC 752. The differences P I − P II, P I − P III, P I − P IV, and P II − P IV were formed for both μ_x and μ_y. Each set of differences for each plate was divided into three subgroups, depending on photovisual magnitude. These groups were $8^m.5$–$9^m.5$, $9^m.6$–$11^m.2$, and $11^m.3$–$12^m.2$. Then, for each of the twenty-four subgroups the quantity $\Sigma v^2/n - 1$ was formed. None of these values of the mean error differed significantly from the mean to indicate a real difference; as a consequence no distinction in weight was made with reference to image diameter, and each plate was given unit weight.

With the aid of these weights the motions from the various plates were combined and are given in Table 2. The columns give, respectively:

1. The author's catalogue number (no previous catalogue has been published).

2 and 3. The co-ordinates, on a system in which $+x$ and $+y$ indicate increasing right ascension and declination, respectively. One unit = 8″.6.

4. The photographic magnitude in Trumpler's system.[2] Most of the magnitudes were kindly supplied by Dr. R. J. Trumpler in advance of publication. For the remainder, interpolations were made on an Eastman 40 plate of the cluster taken with the 24-inch reflector of the Yerkes Observatory. For a few faint stars a slight extrapolation was necessary.

5. The spectral type, kindly communicated by Trumpler.[2] A dash in place of the spectral type indicates that Trumpler supplied the magnitude but gave no spectral type.

6 and 7. The proper motions in x and y expressed in units of 0″.0001/year.

8. The weight of the motions in columns 6 and 7. Unit weight corresponds to a mean error of ±0″.0024 as obtained from the inter-

[2] Private communication.

TABLE 2

THE CATALOGUE

No.	x	y	m_{pg}	Spec.	μ_x	μ_y	Wt.	Membership
1	230	82	10.4	....	+ 3	+ 5	4	1
2	246	103	8.9	....	+ 7	− 85	4	4
3	215	116	9.9	....	+ 25	− 81	4	4
4	243	117	11.6	....	+ 4	+ 20	3	1
5	225	123	10.7	....	+ 9	+ 16	4	1
6	231	126	11.5	....	− 16	+ 1	4	1
7	219	183	11.6	....	− 30	− 18	4	3
8	191	66	9.0	gG5	− 18	+ 4	4	1
9	199	112	11.0	A	+ 9	+ 6	4	1
10	194	118	12.3	—	+ 4	+ 2	2	2
11	205	126	10.9	....	− 22	+ 10	3	2
12	207	132	10.7	....	− 63	+ 22	4	4
13	188	141	9.0	A2	− 6	+ 23	4	1
14	200	148	11.5	....	− 13	+ 22	3	2
15	210	148	10.0	....	− 13	− 8	4	1
16	196	164	10.4	....	− 16	− 10	4	1
17	166	46	10.4	A0	+ 11	+ 6	4	1
18	172	48	10.8	A0	+ 12	+ 12	4	1
19	171	60	12.8	....	−119	− 20	1	4
20	182	63	12.5	....	− 17	− 7	2	2
21	167	90	11.5	G0	−152	+ 48	4	4
22	165	116	12.4	....	− 21	+ 8	2	2
23	173	121	12.2	—	+ 22	+ 7	2	2
24	177	133	12.7	....	− 74	+ 3	1	3
25	176	144	11.3	A	− 9	+ 12	4	1
26	178	139	12.3	....	+ 14	− 63	1	3
27	166	151	10.0	B8	− 3	− 1	4	1
28	182	205	9.2	....	+ 86	+ 69	4	4
29	152	32	9.9	A2	− 5	− 3	3	1
30	154	38	11.4	—	−238	+129	4	4
31	158	44	12.3	....	− 41	− 49	1	3
32	159	55	11.6	—	+ 13	+ 2	2	2
33	149	57	12.3	....	+ 1	−626	3	4
34	148	63	10.7	A3	+ 14	− 7	4	1
35	161	70	11.5	K0	− 11	− 71	4	4
36	154	83	11.5	A	+ 4	0	3	1
37	160	89	10.7	A2	+ 7	+ 2	4	1
38	162	94	11.9	—	+ 21	+ 19	3	2
39	150	98	12.2	—	− 75	− 13	3	4
40	154	106	11.6	A3:	+ 2	− 6	3	1
41	157	106	10.6	A2	+ 8	0	4	1
42	142	119	12.5	....	+ 5	− 28	3	2
43	140	121	11.5	A5	+ 30	+ 10	3	2
44*	138	129	8.8	A2	—	+ 13	2	1
45	142	133	9.7	A1	+ 22	+ 14	4	1

* No. 44: double star, brighter preceding component.

TABLE 2—*Continued*

No.	x	y	m_{pg}	Spec.	μ_x	μ_y	Wt.	Membership
46	148	132	11.6	A:	−5	−7	3	1
47	149	144	9.2	A1	+4	+11	4	1
48	159	143	12.5	....	−66	+19	1	3
49	160	151	11.5	—	−2	+1	3	1
50	161	178	12.4	....	−56	−53	1	3
51	156	180	11.7	—	+16	−10	3	1
52	139	181	8.7	A1	+2	+10	4	1
53	118	48	12.0	—	−144	+18	3	4
54	121	50	11.6	A5:	+18	−34	3	3
55	118	54	11.9	—	−21	0	4	1
56	127	63	11.7	A2	+14	−5	3	1
57	116	68	11.4	A5	+9	+4	2	2
58	119	72	10.2	G8	−1	−15	3	1
59	128	77	12.2	A:	+19	−45	1	3
60	121	84	12.4	K:	+29	−37	4	3
61	129	87	10.9	A1	−1	−17	4	1
62	116	96	12.4	—	+15	−85	2	3
63	113	97	12.6	—	+10	−16	1	2
64	120	103	12.7	K:	−24	+54	2	3
65	134	102	8.9	A1	+14	+2	4	1
66	135	107	9.9	gG0	+1	−4	4	1
67	126	113	9.7	A3	+16	−5	4	1
68	128	115	10.9	A3	+25	−5	4	1
69†	127	118	9.1	F2p	−14	−10	4	1
70	132	122	11.0	A	+5	−14	3	1
71	132	122	10.5	A	+1	+14	3	1
72	137	121	10.1	A2	−14	+8	4	1
73	134	135	12.5	....	+60	−92	1	4
74	111	138	12.5	—	−34	−24	1	3
75	127	147	10.0	A2	+7	0	4	1
76	132	152	9.9	A0	−12	+8	4	1
77	131	192	10.4	—	+15	+5	4	1
78	114	197	12.4	....	−10	−69	1	3
79	131	232	12.2	....	−43	−67	1	4
80	98	92	11.4	A3	+10	−12	3	1
81	104	94	9.8	A1	+18	+4	4	1
82	107	105	9.3	A1	+10	+16	4	1
83	97	125	9.6	A1	−11	−1	4	1
84	99	126	12.5	....	−159	−58	1	4
85	99	132	12.0	—	+31	−5	3	2
86	103	146	10.9	G0	−238	−110	4	4
87	103	147	12.4	—	−235	−111	3	4
88	87	154	10.5	A:	+29	−7	4	2
89	104	161	12.8	....	−180	−158	1	4
90	87	174	8.4	A1	−11	+6	4	1

† No. 69: according to Trumpler, the F2p spectrum is probably composite A+G.

TABLE 2—*Continued*

No.	x	y	m_{pg}	Spec.	μ_x	μ_y	Wt.	Membership
91	97	192	12.7	....	−7	+1	1	2
92	103	201	12.0	....	−9	−20	2	2
93	106	205	12.3	....	+78	−39	1	4
94	90	207	10.5	....	−8	+17	4	1
95‡	90	209	8.8	A1	+8	−13	4	1
96	87	220	10.9	....	+1	+13	2	2
97	104	220	12.0	....	+40	−3	1	2
98	82	46	10.6	A0	+4	−19	4	1
99	66	71	9.8	A2	−8	−4	4	1
100	78	116	10.1	A2	+12	−9	4	1
101	64	128	9.2	A1	+15	−27	4	2
102	67	155	12.0	—	−80	−142	2	4
103	71	172	9.1	A1	−2	−6	4	1
104	68	185	11.9	....	+10	+6	2	2
105	80	212	11.0	....	+45	−93	2	4
106	37	66	10.7	....	−36	−9	4	3
107	54	85	11.2	A2	+12	−5	4	1
108	42	86	10.7	....	+59	−21	4	4
109	31	106	11.7	....	+73	−104	2	4
110	49	127	11.7	....	+10	−93	3	4
111	47	157	11.8	....	+18	−17	3	1
112	59	191	12.4	....	+13	+59	1	3
113	59	191	12.4	....	+20	+92	1	4
114	51	194	10.9	....	−86	−45	4	4
115	53	202	9.3	....	+10	+5	2	2

‡ No. 95: double star, north following component.

nal agreement between the plates. A number of faint stars on P I did not occur on any of the other three plates; and, although no direct determination of their weight could be made, it was considered reasonable to give them the same weight as the remainder of the stars on P I, i.e., unit weight.

9. A measure of the probability that the star is a cluster member. The figure 1 indicates that the star has a high probability of being a cluster member; 2 indicates that the probability is lower but that the star has a reasonable chance of being a member; 3 indicates that the star is probably not a member; and 4 signifies that the star is definitely not a cluster member. This assignment of membership will be discussed in the next section.

III. DISCUSSION OF THE RESULTS

The data given in Table 2 are complete down to photovisual magnitude 12.0, within a radius of 15′ of star No. 72. As a result of the close proximity of the image of a fainter star to the image of No. 44, it was possible to measure its motion only in the y-direction. As a result of its motion ($\mu_\delta = 0\overset{''}{.}0013$/year) and its position in the spectrum-magnitude diagram ($\log T_e = 4.04$ and $m_{\text{bol}} = 8.2$), No. 44 may provisionally be considered as being a cluster member.

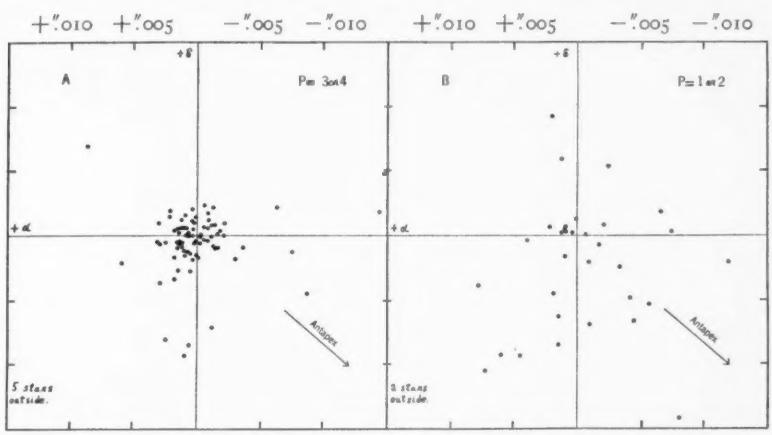


FIG. 1

The weights assigned to the individual stars range from 1 to 4. A weight of 4 corresponds to a mean error of $\pm 0\overset{''}{.}0012$/year in either co-ordinate. Figure 1, *A*, is a plot of all stars with weight of 3 or 4, and in Figure 1, *B*, are plotted all stars with weight 1 or 2.

The determination of the probability of membership in the cluster was made in the following manner. The average weight of the stars in Figure 1, *A*, corresponds to a mean error of $\pm 0\overset{''}{.}0013$/year. Three circles, of radii 2, $2\sqrt{2}$, and 4 times the mean error just given, were drawn with their centers at the center of gravity of the concentration. Stars within the first circle were assigned to class 1, stars between the first and second circles to class 2, those between the second and third circles to class 3, and the stars outside of the third circle to class 4.

The average weight of the stars in Figure 1, *B*, corresponds to a mean error of $\pm 0\overset{''}{.}0021$/year. About the center of gravity two circles

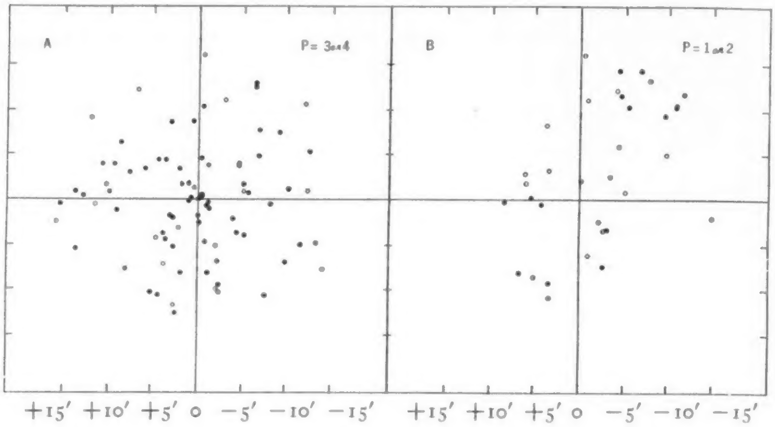


FIG. 2

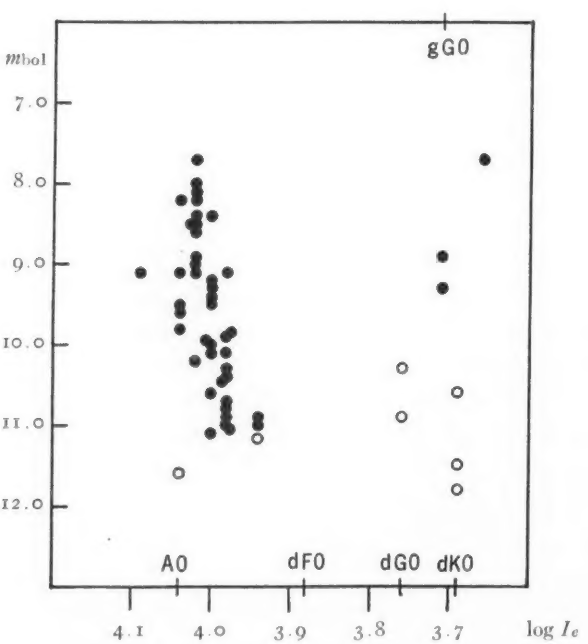


FIG. 3

were drawn with radii 2 and 4 times this mean error. No class-1 membership was assigned to any star in this group. Class-2 mem-

bership was assigned to the stars inside the first circle, and class-4 membership to those outside the second circle.

The number of stars with a membership class of 1 or 2 is 74.

Figure 2, *A*, is a plot of the position in the cluster of all stars whose weight is either 3 or 4, and Figure 2, *B*, is a plot of all whose weight is either 1 or 2. In both figures the filled circles correspond to stars with a class membership of 1 or 2.

Figure 3 is a plot of the spectrum-magnitude diagram. The filled circles represent stars with a membership class of 1 or 2, and the open circles are those of class 3 or 4. The abscissas are log T_e, and

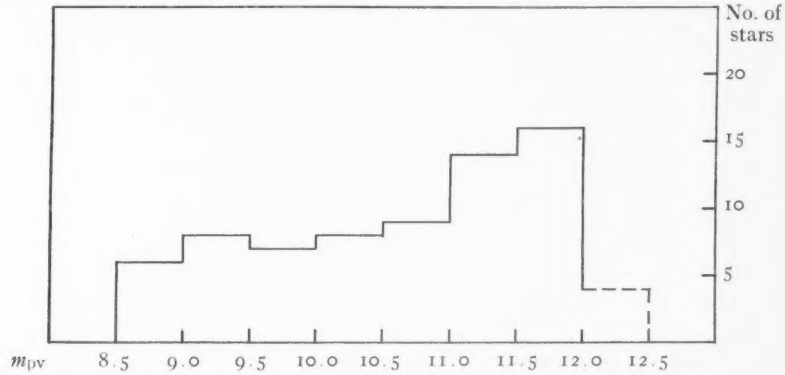


FIG. 4.—Luminosity function for the cluster members

the ordinates are apparent bolometric magnitudes. The reduction of Trumpler's spectra and photographic magnitudes to this new system was carried out with the aid of the table given by Kuiper in his discussion of the hydrogen content of clusters.[3]

Figure 4 gives the luminosity function of the cluster with the photovisual magnitude as the abscissa. The diagram is complete for the area with a radius of 15′ around star 72 and down to about $m_{pv} = 12.0$. The distance modulus of NGC 2548 is about 8^m6.[4]

IV. THE PROPER MOTION OF THE CLUSTER

Since there are only a few faint stars in the cluster, and because none of the measured stars occur in either the *Boss General Catalogue* or Schorr's *Eigenbewegungs Lexikon*, no attempt has been made to determine the absolute proper motion of the cluster.

[3] *Ap. J.*, **86**, 176, 1937. [4] Trumpler, *Lick Obs. Bull.*, No. 420, 157, 1930.

The writer wishes to express his sincere thanks to Dr. Trumpler for having communicated in advance of publication his spectral types and magnitudes for this cluster. Grateful acknowledgment is made to Dr. Struve for having enabled the writer to continue this investigation after he had left the Yerkes Observatory to teach at Wilson College. Also, I wish to express my thanks to Dr. Kuiper for his general guidance.

YERKES OBSERVATORY
June 1939

THE ULTRAVIOLET SPECTRA OF A AND B STARS*

OTTO STRUVE

ABSTRACT

This paper contains (*a*) the measured wave lengths of 350 absorption lines in the A stars α Cygni, α Lyrae, η Leonis, and α^1 Canum Venaticorum, between λ 3227 and λ 3957; (*b*) the measured wave lengths of 198 lines in the O and B stars 10 Lacertae, τ Scorpii, β Cephei, γ Pegasi, and 55 Cygni. The identifications of the star lines with laboratory lines are given for each group of stars.

The Cassegrain spectrograph of the McDonald Observatory is equipped with two quartz prisms made by Bausch and Lomb and is primarily intended for work in the near ultraviolet region of the spectrum. The specifications of the prisms are as follows:

First Cornu prism:	Angle, 62°11′
	Length of edge, 80 mm
	Length of face, 130 mm
Second Cornu prism:	Angle, 62°11′
	Length of edge, 80 mm
	Length of face, 140 mm

The prisms are set at minimum deviation for λ 3933; the total deviation is 90°.

The collimator has a focal length of 1 m and consists of two pieces of UV glass—UBK5 and UZK5, respectively. Its aperture is 76 mm. The focal length of the camera lens is 50 cm. It consists of four pieces of UV glass, having an aperture of 86 mm. Both lenses were designed by Dr. F. E. Ross; they were made by Mr. J. W. Fecker at Pittsburgh, Pennsylvania. Two additional lenses, of quartz, are not yet available. The ultraviolet limit of the transparency of the UV lenses is about λ 3250. At λ 3933 the linear dispersion with the UV camera is about 40 A/mm. At λ 3250 it is about 20 A/mm.

The mechanical parts of the spectrograph were designed by Dr. G. W. Moffitt and were built under his direction by Mr. Charles Ridell in the instrument shop of the Yerkes Observatory. A descrip-

* *Contributions from the McDonald Observatory, University of Texas,* No. 13.

tion of the instrument by Dr. Moffitt will appear in No. 1 of the *Contributions of the McDonald Observatory*.

In May and June, 1939, the spectrograph was used to obtain nearly 100 widened spectra of bright A- and B-type stars having sharp lines. The observations were made in collaboration with Dr. A. Unsöld, who will use the material for a study of line intensities. The following plates were chosen for measurement:

α Cygni (A2 supergiant)	Four plates taken on May 26, June 1, and June 2
α Lyrae (Ao dwarf)	Four plates taken on May 25, June 1, and June 2
η Leonis (Ao giant)	Two plates taken on May 26 and 27
α Can. Ven. (Ao dwarf)	Two plates taken on May 26
10 Lacertae (O9 dwarf)	One plate taken on May 26
τ Scorpii (B1 dwarf)	Three plates taken on May 24, 25, and 26
β Cephei (Bo dwarf)	One plate taken on May 27
γ Pegasi (B2 dwarf)	One plate taken on June 1
55 Cygni (B2 supergiant)	One plate taken on June 1

All plates are on Eastman Process emulsion. During measurement two similar spectra were kept in the machine simultaneously, although only one was actually being measured. The other plate served as a comparison spectrum and was used to verify the reality of very faint lines.

Tables 1 and 2 contain the results of the measurements. The work on the A stars is a continuation of Morgan's study of the spectra of A stars in the ordinary photographic region.[1] My measurements overlap his near λ 3900, and the agreement is very satisfactory. The near ultraviolet region of A and B stars has already been investigated by Swings and Désirant,[2] who used plates taken with the Yerkes autocollimating spectrograph at the Perkins Observatory, and by Marshall,[3] who measured spectrograms taken with the light-glass spectrograph at Ann Arbor. The spectrum of α Cygni has been completely measured in the ultraviolet region by W. H. Wright[4] and A. B. Wyse,[5] at the Lick Observatory. The agreement of my results with those of Wyse's compilation is very satisfactory. My identifications and assignments of the major contributors to the lines were made quite independently, and there occur a few—rather un-

[1] *Pub. Yerkes Obs.*, **7**, Part III, 1935.

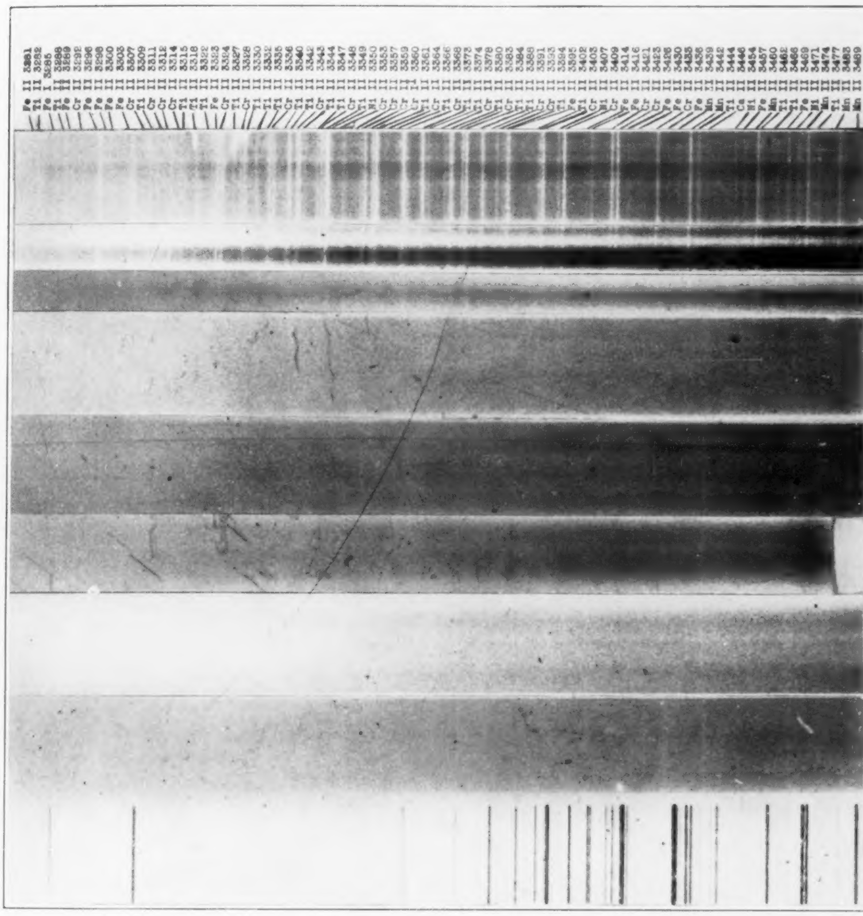
[2] *Ap. J.*, **83**, 35, 1935.

[3] *Pub. Obs. U. of Michigan*, **5**, No. 12, 1934.

[4] *Lick Obs. Bull.*, **10**, 100, 1921.

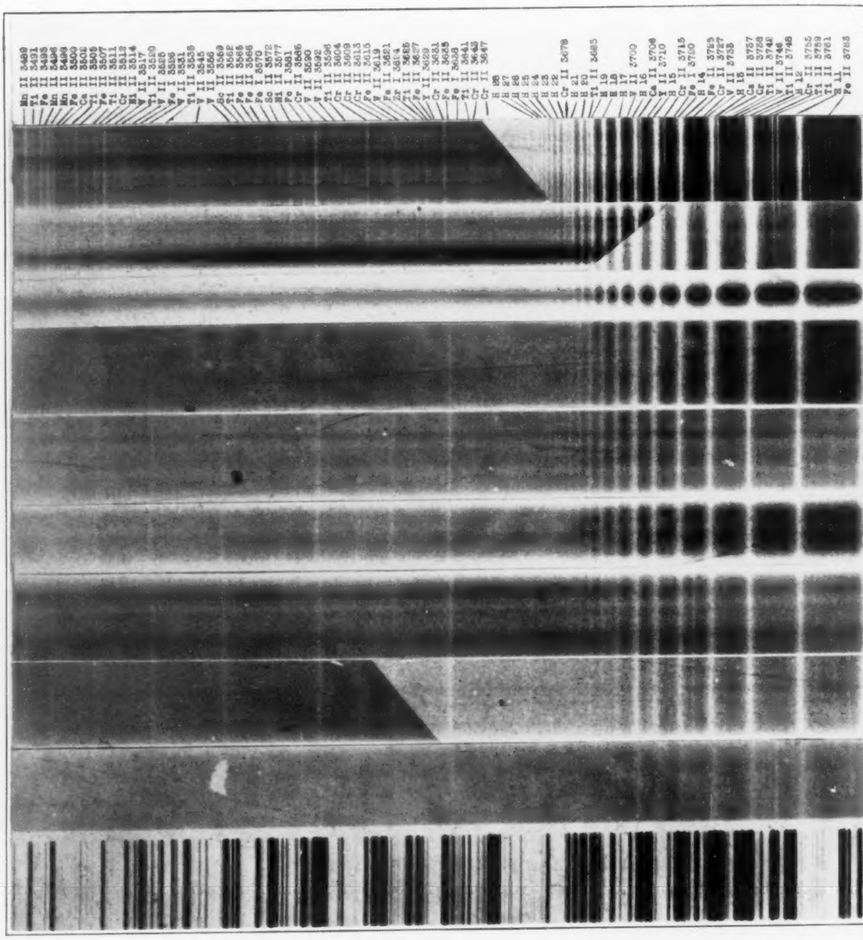
[5] *Lick Obs. Bull.*, **18**, 129, 1938.

PLATE VIII



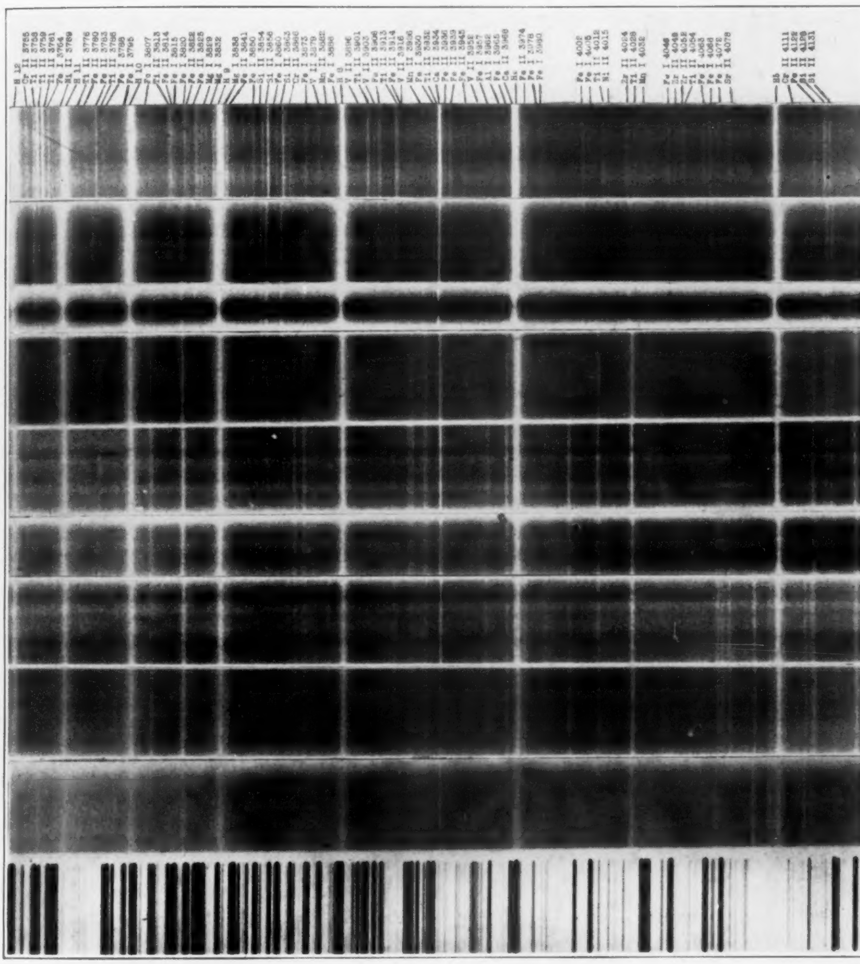
SPECTRA OF A AND B STARS

PLATE IX



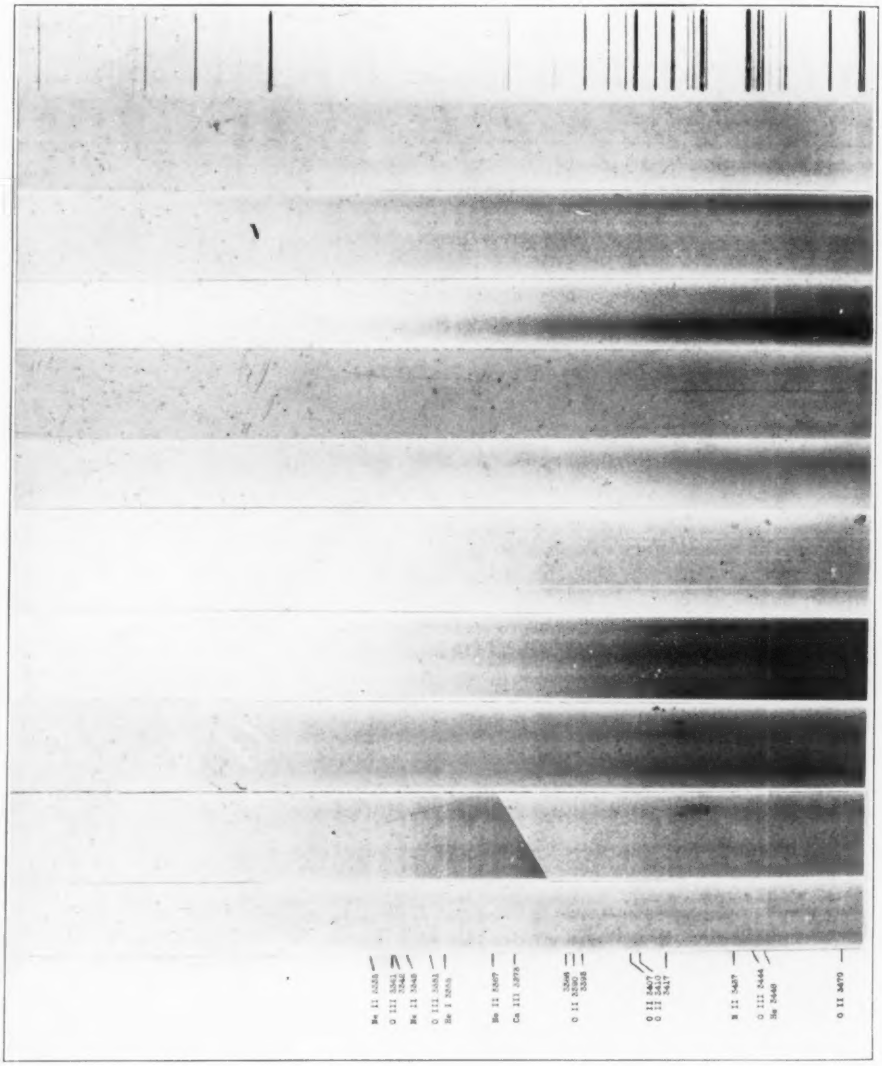
SPECTRA OF A AND B STARS

PLATE X



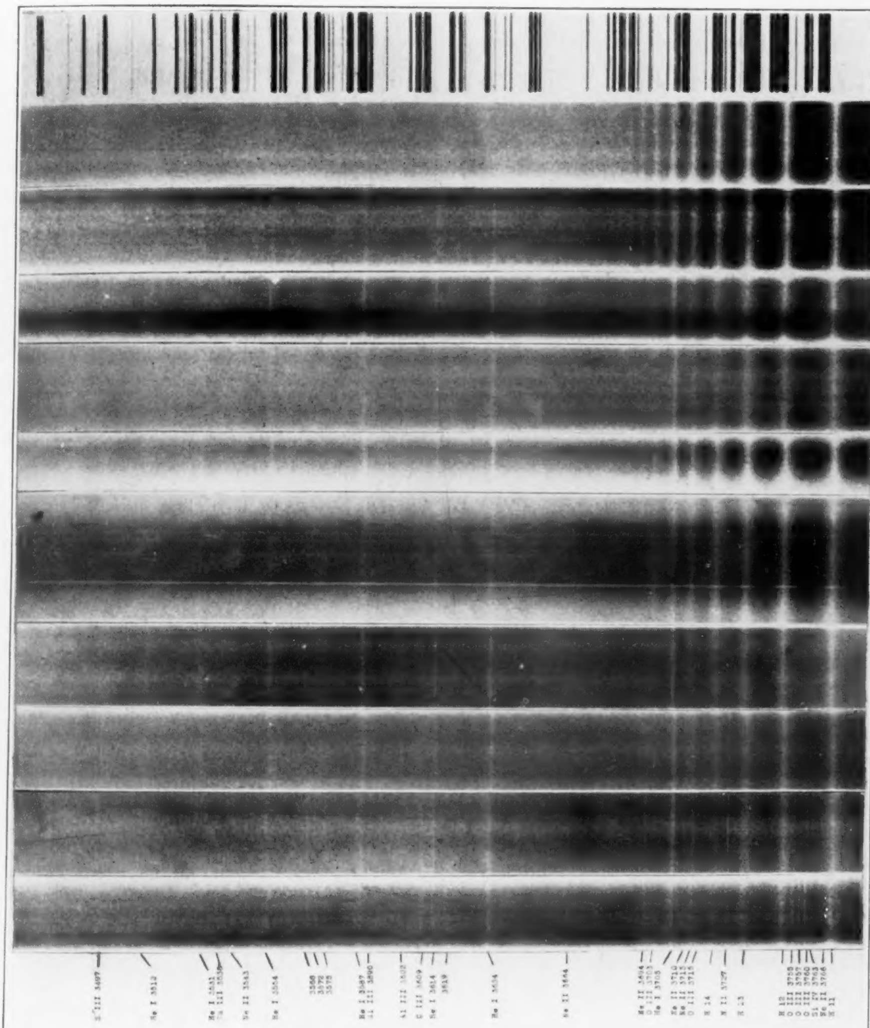
SPECTRA OF A AND B STARS

PLATE XI



SPECTRA OF B STARS

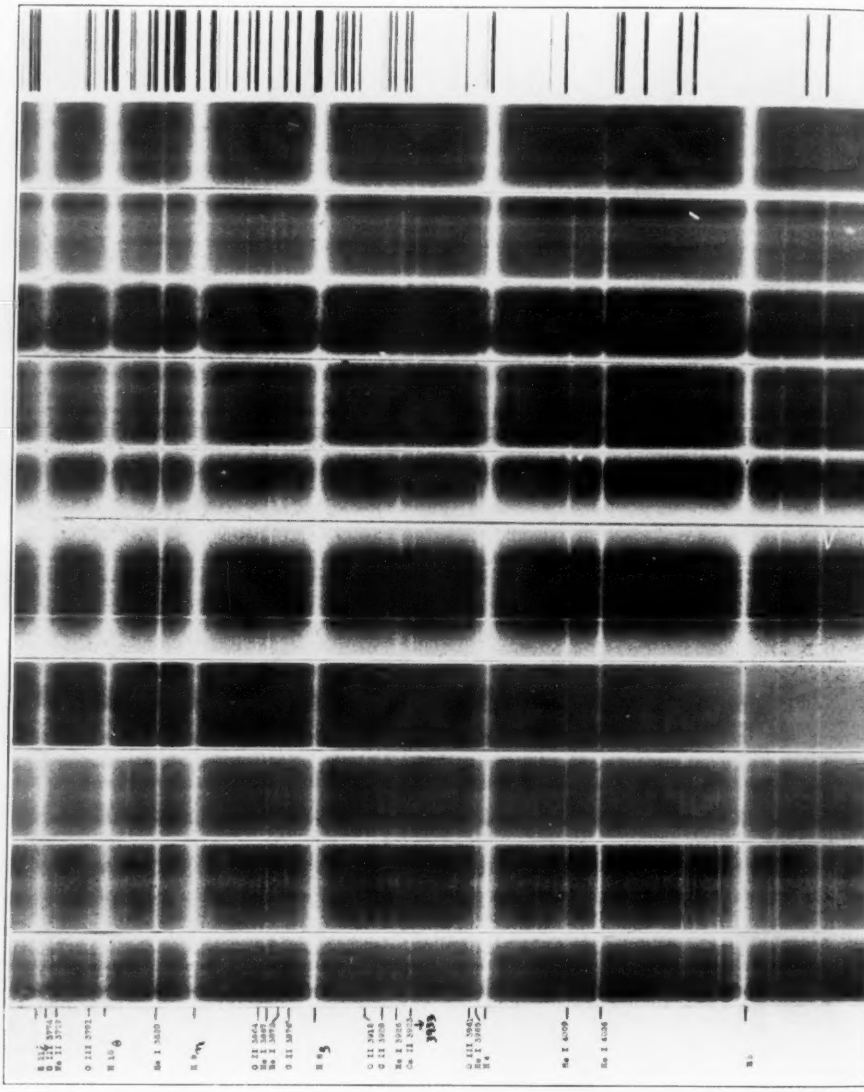
PLATE XII



B5(d)
B3(d)
B3(d)
B3(d)
B3(d)
B3(d)
B2(d)
B1(d)
B0(d)
Oe5(d)

SPECTRA OF B STARS

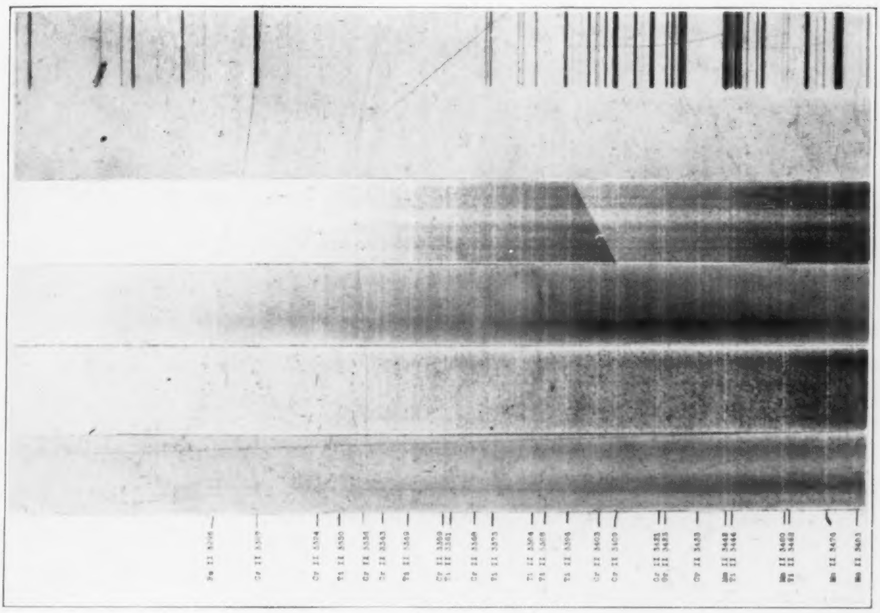
PLATE XIII



ζ Draconis	B5(d)
η Lyrae	B3(d)
ζ Cassiopeiae	B3(d)
θ Ophiuchi	B3(d)
102 Herculis	B3(d)
ι Herculis	B3(d)
γ Pegasi	B2(d)
β Cephei	B1(d)
τ Scorpii	B0(d)
10 Lacertae	Oe5(d)

SPECTRA OF B STARS

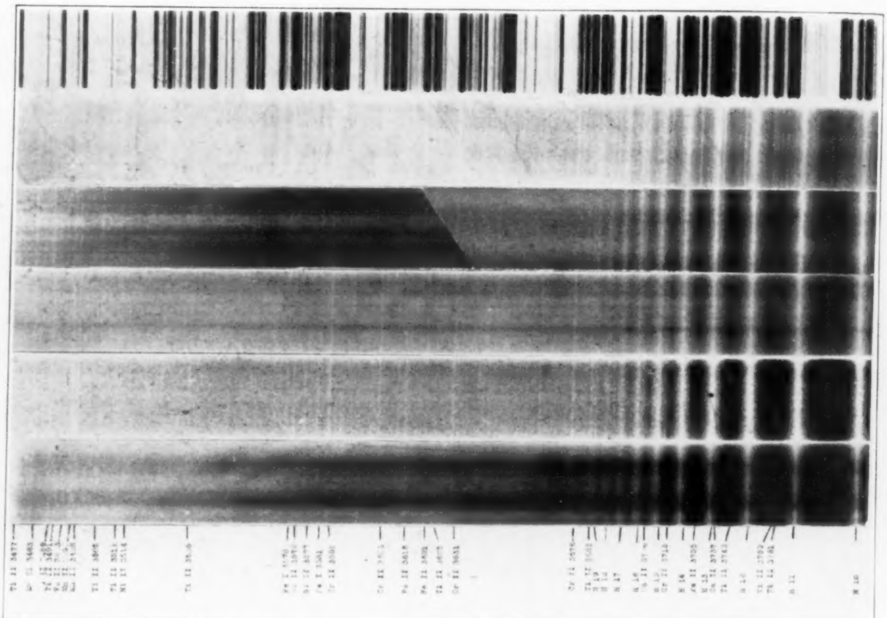
PLATE XIV



Serpentis	A2(d)
Lyrae	Ao(d)
Can. Ven.	Aop
Herculis	Bop
Corvi	B8(d)

SPECTRA OF A AND B STARS

PLATE XV



ϵ Serpentis — A2(d)

α Lyrae — A0(d)

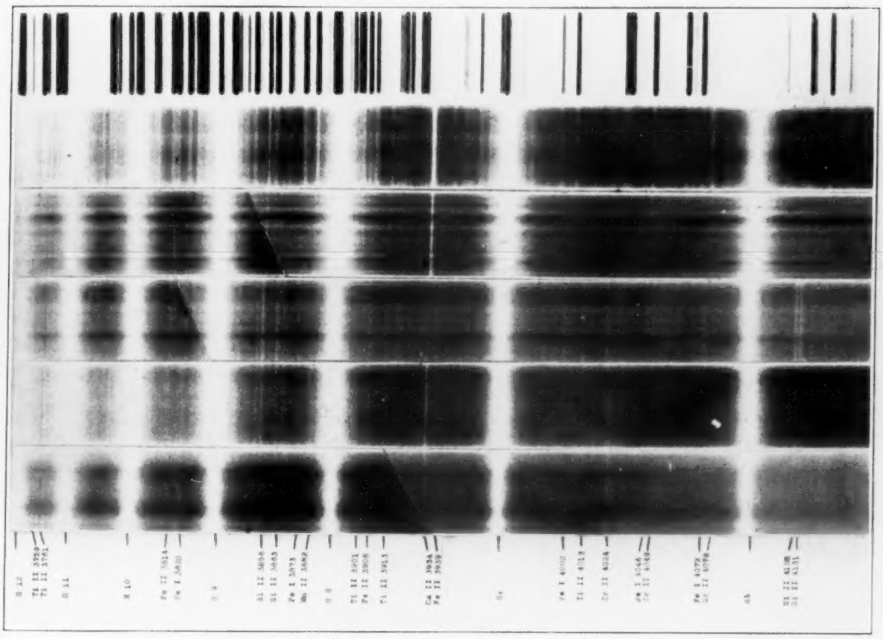
α^2 Can. Ven. — A0p

ϕ Herculis — B0p

γ Corvi — B8(d)

SPECTRA OF A AND B STARS

PLATE XVI



ε Serpentis		A2(d)
α Lyrae		A0(d)
² Can. Ven.		A0p
Herculis		B9p
Corvi		B8(d)

SPECTRA OF A AND B STARS

important—differences in the assignments of the various laboratory lines. My plates show considerably more lines than those listed by Wright, as should, of course, be expected in consequence of the fine optical quality of the new Cassegrain spectrograph. Wyse has already pointed out that a study of the spectrum of α Cygni with relatively small dispersion cannot be considered as final. But the star is so important in many spectrographic investigations that a fairly complete list of lines observable with our instruments will be valuable. Moreover, the lines of α Cygni are by no means narrow. They look as though they are appreciably broadened by turbulence. Hence, the blending of lines is essentially intrinsic in character. Relatively little will be gained by the use of larger dispersion.[6] The ultraviolet spectrum of α Lyrae has been investigated by R. W. Shaw[7] from plates taken with a quartz spectrograph at the Lowell Observatory. In the region λ 3000 to λ 3480 the wave lengths of 118 lines were determined, but no lines were seen between λ 3480 and the limit of the Balmer series. My plates confirm the general weakening of the lines in the region of strong continuous hydrogen absorption, in α Lyrae and in other A stars. But some 40 lines were measured in the interval $\lambda\lambda$ 3480–3647. The equivalent widths of these lines will yield quantitative information concerning the amount of continuous absorption at different wave lengths.

The ultraviolet spectra of the B stars have been recently investigated at the Mount Wilson Observatory: Adams and Dunham[8] have given lists of wave lengths ranging from λ 3023 to λ 3664, for several stars; Sanford[9] has measured the ultraviolet and violet spectra of τ Scorpii and has identified several new *Ne* II lines. The new measurements extend my previous work in the photographic region of the spectra of B stars[10] and partly overlap with the work of Kühlborn.[11]

In making the identifications Miss Moore's *A Multiplet Table of Astrophysical Interest* (Princeton, 1933) was extensively used. I am

[6] An extension of this work with the coudé spectrograph of the McDonald Observatory is being planned.

[7] *Ap. J.*, **82**, 87, 1935.

[8] *Ap. J.*, **87**, 102, 1938.

[9] *Pub. A.S.P.*, **50**, 244, 1938.

[10] *Ap. J.*, **74**, 225, 1931; **77**, 321, 1933.

[11] *Veröff. Berlin-Babelsberg*, **12**, Part I, 1938.

very deeply indebted to Miss Moore for unpublished lists of important lines of *Fe* II, *Cr* II, and *V* II. To Dr. P. Swings I am grateful for a long list of unpublished wave lengths of *Fe* III. The original literature was frequently consulted, and many additional identifications could be made by going to the sources.

The intensities of the star lines in Tables 1 and 2 are rough estimates and are intended solely to serve as a guide in making proper identifications. Intensity 1 means that the line was visible on the two plates kept in the measuring machine. These lines are regarded as certain. Intensity 1–0 means a line which was clearly visible on only one plate. The vast majority of these lines is real, but a few may be spurious. Intensity 0–1 indicates a line which is visible on one plate and which is regarded as suspicious. The letters "n," "nn," "nr," and "nv" mean, respectively: nebulous, very nebulous, nebulous and shaded toward the red, and nebulous and shaded toward the violet.

The precision of the measurements varies greatly with the intensity of the lines. All lines of intensity 2 and stronger are measured to within a few hundredths of an angstrom unit in α Cygni, the star which yielded the most precise results. Very faint lines, or nebulous lines, may be in error by a large fraction of an angstrom unit. Among the B stars the most trustworthy wave lengths are those of τ Scorpii. But even these may occasionally be in error by as much as 0.5 A, or even more.

The wave lengths of the laboratory lines are essentially those of Miss Moore's table. Accordingly, solar values are given where these are listed by her, and laboratory values are given only for those elements which are weak or absent in the spectrum of the sun. The intensities of these lines are those determined in the laboratory. They are, of course, comparable only for lines of any given atom or ion. The most important contributor or contributors of the star lines are designated by the plus sign (+) for α Cygni. Caution should be used in applying these results to other A stars. For the B stars the contributors vary so much from star to star that no attempt has been made to assign them. Some laboratory lines which appear to be too weak to account for the star line, or which differ appreciably in wave length, are given in parentheses.

TABLE 1

A STARS

α CYGNI		α LYRAE		η LEONIS		α CAN. VEN.		IDENTIFICATION			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3227.86	7	......	......	......	......	......	......	*Fe* II	3227.73	13	......
3229.26	7	......	......	......	......	......	......	*Ti* II	3228.62	30	......
								Ti II	3229.21	40	+
								Ti II	3229.43	35	......
3232.18	8	......	......	......	......	......	......	*Fe* II	3231.70	5	......
								Ti II	3232.29	30	+
								Fe II	3232.79	7	......
3234.40	7	......	......	......	......	......	......	*Cr* II	3234.06	30	......
								Ti II	3234.52	75	......
3236.19	6	......	......	......	......	......	......	*Ti* II	3236.13	20	......
								Ti II	3236.59	70	+
3237.46	4	......	......	......	......	......	......	*Fe* II	3237.40	5	......
								Fe II	3237.82	8	+
								V II	3237.88	350	......
3239.29	4	......	......	......	......	......	......	*Cr* II	3238.77	25	......
								Ti II	3239.05	60	+
								Ti II	3239.67	30	......
3241.89	4	......	......	2.15	3	......	......	*Fe* II	3241.68	2	......
								Ti II	3242.00	60	+
3243.77	3	......	......	3.67	3	......	......	*Fe* II	3243.72	8	......
3247.21	5	......	......	6.94	2	......	......	*Fe* II	3247.17	9	+
								Fe II	3247.39	3	......
3248.63	2	......	......	......	......	......	......	*Ti* II	3248.61	50	......
3249.61	1	......	......	......	......	......	......	*Fe* II	3249.66	4	+
								Fe II	3249.91	1	......
3251.79	2	......	......	......	......	......	......	*V* II	3251.87	200	......
								Ti II	3251.90	30	+
3252.79	2	......	......	2.63	1	......	......	*Ti* II	3252.89	40	......
3254.19	3	......	......	4.78	1	......	......	*Ti* II	3254.26	30	......
3255.93	4	......	......	5.91	2	......	......	*Fe* II	3255.88	8	......
3258.87	6	9.31	3	8.92	3	......	......	*Fe* II	3258.77	10	+
								Cr II	3258.77	8	......
								Fe II	3259.05	10	+
3261.50	4	1.91	4	1.58	1	......	......	*Ti* II	3261.58	60	......
3264.34	4 n	......	......	4.34	2 n	......	......	*Ti* II	3263.69	4	......
								Cr II	3264.26	12	+
								Fe II	3264.77	pr	......
3266.90	3	......	......	7.11	1	......	......	*Fe* II	3266.94	4	+
								Fe II	3267.04	3	......
3268.09	2	......	......	8.32	1	......	......	*V* II	3267.71	1000	+
								Cr II	3268.48	4	......
								Fe II	3268.51	3	+
3269.99	2	......	......	9.74	1	......	......	*Fe* II	3269.77	2	......
								Cr II	3269.77	7	......
								Cr II	3270.14	20	+

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3271.55	4 n	......	......	1.82	1	......	......	*V* II	3271.12	1200	......
								Ti II	3271.67	25	+
								Ti II	3272.10	25	+
3273.49	1	......	......	......	......	......	......	*Fe* II	3273.50	3	......
3276.35	3	......	......	6.04	1	......	......	*Cr* II	3275.92	3	......
								V II	3276.12	1500	+
								Fe II	3276.61	5	+
3277.23	5	......	......	7.32	2	......	......	*Fe* II	3277.35	9	......
3279.25	1	......	......	9.39	1–0 n	......	......	*Ti* II	3278.30	30	+
								Fe II	3279.65	2	+
								V II	3279.84	300	......
3281.21	4	......	......	1.22	3	......	......	*Fe* II	3281.29	7	......
3282.41	2	......	......	2.43	1	......	......	*Ti* II	3282.34	25	+
								V II	3282.53	150	......
								Cr II	3283.04	8	......
3285.41	3	......	......	5.23	1	......	......	*Fe* II	3285.42	3	+
								Cr II	3285.96	8	......
3287.65	3	7.81	3	7.66	2	8.01	2	*Fe* II	3287.47	1	......
								Ti II	3287.67	40	+
								Cr II	3288.04	5	......
3289.39	3	......	......	9.46	2	9.24	1	*Fe* II	3289.35	7	+
								V II	3289.39	100	......
3291.74	2	......	......	1.86	3	1.85	2	*Cr* II	3291.75	20	......
3293.25	1	......	......	3.13	1–0	......	......	(*V* II	3293.15	50)	......
								(*Ti* II	3293.48	pr)	......
3295.57	10	5.67	4	5.57	4 n	5.64	5	*Fe* II	3295.24	4	......
								Cr II	3295.42	30	+
								Fe II	3295.81	6	+
......	......	......	......	......	......	6.91	1	*Fe* II	3296.83	2	......
3298.01	1 n	......	......	8.11	1	8.08	1	*Fe* II	3297.89	5	+
								V II	3298.74	130	......
3300.01	1–0	......	......	......	......	......	......	*Fe* II	3299.77	1	......
......	......	......	......	1.51	1	......	......	*Cr* II	3301.21	5	......
......	......	......	......	2.86	1	......	......	*Fe* II	3302.86	4	......
3303.19*	8	......	......	3.47	2	......	......	*Fe* II	3302.86	4	......
								Fe II	3303.47	4	......
3307.10	3	......	......	7.06	2	6.94	2	*Cr* II	3306.95	12	......
								Cr II	3307.04	30	+
3308.76	2	......	......	......	......	9.15	1	*Cr* II	3308.15	5	......
								Ti II	3308.82	8	+
3310.56	2	......	......	......	......	......	......	*Cr* II	3310.65	18	......
3312.08	4	......	......	2.11	2	2.26	1	*Cr* II	3311.93	20	+
								Cr II	3312.18	18	+
								Fe II	3312.71	1	......

* This line is blended in α Cygni but is resolved in η Leonis.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3314.03	2	4.16	1	....	......	4.37	2 n	*Fe* II *Cr* II *Cr* II	3314.00 3314.06 3314.57	1 6 15	....
3315.33	2	5.52	1	5.01	1 n	....	......	*Cr* II *Ti* II *Fe* II	3315.29 3315.33 3315.52	4 10 pr	...... + ...
3317.99	3	7.92	2	8.32	1–0 n	....	......	*Ti* II (*Fe* II	3318.03 3318.61	10 pr)	....
3319.97	1	9.97	1	....	......	....	......	......	....	......	......
3321.75	3	1.72	2	1.81	1	....	......	*V* II *Ti* II	3321.54 3321.71	150 25	...... +
3322.97	6	3.05	4	3.12	4	2.88	6	*Cr* II *Ti* II *Fe* II *Ti* II	3322.69 3322.93 3323.07 3323.40	7 75 8 pr	...... + + ...
3324.20	5	4.20	2	4.45	3	4.25	4	*Cr* II *Cr* II *Cr* II	3324.05 3324.10 3324.34	20 10 12	+ ...... +
....	......	5.67	1	....	......	....	......	*Fe* II	3325.01	1	......
3326.85	3	6.94	2	6.94	1 n	6.82	1	*Ti* II	3326.78	20	......
3328.34	2	8.32	1	8.39	1	8.44	1	*Cr* II	3328.35	12	......
3329.51	5	9.42	2	9.62	1	9.47	3	*Ti* II	3329.5	70	......
3332.07	4	1.97	2 n	2.24	1	2.33	1 n	*Ti* II	3332.11	30	......
3335.22	6	5.17	3	5.26	3	5.41	3	*Ti* II *Cr* II *Cr* II	{3335.17 3335.22} 3335.28 3335.46	40 30 20	....
3336.26	5	6.52	1	6.49	2	6.38	3	*Cr* II	3336.32	35	......
3337.51	1	....	......	....	......	7.64	1	*V* II *Ti* II	3337.84 3337.85	200 2	...... +
3338.53	1	8.37	2	8.63	1–0	8.84	2	*Fe* II	3338.52	3	......
3339.98†	7 nv	9.64†	1	40.00†	4 n	39.90	4	*Cr* II *Si* II *Cr* II *Ti* II	3339.80 3339.84 3339.90 {3340.333 3340.392}	50 3 15 35	+ .... +
....	......	40.27	2	....	......	....	......	*Ti* II	{3340.33 3340.39}	35	......
3341.92	5	1.83	3	1.92	3	....	......	*Ti* II	3341.84	100	......
3342.53	5	2.56	2	2.76	2	2.28‡	3 n	*Cr* II	3342.51	50	......
3343.68	3	3.91	2	4.19	1	....	......	*Ti* II	3343.78	10	......
3346.73	4	6.98	2	6.81	1	6.89	2	*Ti* II	3346.7	15	......
3347.89	5	8.01	1	7.88	2	7.84	3	*Cr* II	3347.83	30	......

† Blended in α Cygni with following line; also in η Leonis, but is resolved in α Lyrae.

‡ Blended in α Canum Venaticorum with *Ti* II 3341.84.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3349.22	10	9.29	10	9.35	6	9.19	4 n	*Ti* II *Ti* II *Cr* II *Ti* II	3348.91 3349.00 3349.34 {3349.39 3349.47}	10? 75 6 125	...... + ... +
3350.46	1	......	......	......	......	......	......	*Ni* II	3350.41	5	......
3352.15	1	2.50	1 n	......	......	......	......	*Ti* II	3352.07	5	......
3353.15	3 n	3.53	2 n	3.18	1	3.10	4	*Cr* II *Sc* II	3353.12 3353.74	18 20	+ ......
3355.65	1 n	......	......	6.18	1–0	5.46	1–0	*Fe* II	3356.26	2	......
3357.41	2	7.15	1–0	7.58	1	7.51	2	*Cr* II	3357.40	4	......
3358.51	5	8.72	3	8.57	4	8.59	3	*Fe* II *Cr* II	3358.25 3358.50	3 75	...... +
3360.27	4	0.29	1	0.32	3	0.29	3	*Fe* II *Cr* II	3360.10 3360.30	3 100	...... +
3361.36	9	1.39	4	1.36	4	1.69	2	*Ti* II *V* II *Cr* II	3361.19 3361.51 3361.77	125 60 20	+ ....
......	......	2.60	0–1	......	......	......	......	(*Ti* II	3362.65	1)	......
3363.69	2 n	......	......	3.83	1 n	......	......	*Cr* II	3363.71	6	......
3365.08	0–1	......	......	......	......	4.79	1	......	......	......	......
3366.23	1	6.24	1–0	......	......	......	......	*Ti* II *Fe* II	3366.18 3366.96	8 3	+ ......
3368.01	7	7.87	3	8.03	6	7.79	2 n	*Cr* II *Cr* II	3367.42 3368.05	10 150	...... +
3368.97	2	......	......	9.32	1–0	9.35	2 n	*Cr* II *Cr* II *Ti* II *Fe* II	3368.73 3369.05 3369.22 3369.35	8 12 2 3	...... + ... +
3370.82	1	0.77	2	......	......	0.82	1	*Fe* I	3370.80	10	......
3372.06	0–1	2.22	1	......	......	2.39	2 n	*Cr* II *Ti* II	3372.13 3372.23	10 10?	....
3372.73	7	2.83	4	2.79	3	......	......	*Ti* II	{3372.77 3372.86}	100	......
3374.11	3	4.98§	1–0	3.95	1	4.50	1 n	*Ni* II *Ti* II	3373.98 3374.35	4 8	....
3376.39	2	......	......	6.44	0–1	6.45	2	*Cr* II	3376.27	5	......
3377.27	1–0	......	......	......	......	......	......	*Cr* II *Cr* II	3376.72 3377.36	3 2	....
3378.38	4	......	......	8.32	2	8.29	1	*Cr* II	3378.34	20	......
3379.95	6 n	80.25	3 n	79.83	4 n	79.91	3 n	*Cr* II *Cr* II *Ti* II	3379.37 3379.82 {3380.26 3380.31}	25 60 30	...... + +

§ In α Lyrae only *Ti* II.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3381.23	1–0	....	......	1.38	1–0	1.45	1	*Fe* II	3381.00	4	......
3382.71	4	2.60	2	2.78	3	2.60	3	*Cr* II	3382.68	60	......
3383.78	6	3.80	5	3.85	5	4.00	1	*Ti* II	3383.77	125	......
....	......	6.15	1 n	5.82	1–0	5.61	1–0	(*Fe* II	3386.45	1)	......
3387.87	5	7.99	3	7.79	3	7.89	3	*Cr* II *Ti* II *Fe* II	3387.73 3387.85 3388.13	3 50 2	...... + ...
....	......	....	......	8.77	0–1	....	......	*Ti* II	3388.76	8	......
3391.47	4	1.26	2 n	1.39	2	1.37	1	*Fe* II *Cr* II	3391.30 3391.42	1 60	...... +
3392.97	3	....	......	3.18	1	3.19	1	*V* II *Cr* II	3392.66 3393.00	50 25	...... +
3394.37	6 nv	4.56	3 n	4.40	3 n	4.45	2	*Cr* II *Cr* II *Ti* II	3393.86 3394.32 3394.55	30 30 40	....
3395.74	2	5.98	1	....	......	5.87	1	*Fe* II *Cr* II	3395.34 3395.62	4 15	....
3397.17	1–0	....	......	....	......	....	......	(*Lu* II *Ni* II	3397.06 3397.84	150) 1	....
3398.37	2	....	......	8.36	1	....	......	*Fe* II	3398.36	4	......
3399.94	1 n	....	......	9.45	1–0	....	......	*Cr* II	3399.54	10	......
3402.46	3	2.18	1–0	2.36	1	2.29	1	*Ti* II *Cr* II	3402.42 3402.43	8 20	....
3403.30	6	3.49	3	3.43	4	3.31	3	*Cr* II	3403.32	100	......
3405.13	1	....	......	5.22	0–1	5.17	0–1	(*Ti* II	3404.96	1)	......
3407.22	4	7.14	2	7.28	2	7.30	1	*Ti* II *Ni* II	3407.21 3407.32	3 8	...... +
3408.83	6	8.89	3	8.83	4	8.83	3	*Cr* II	3408.77	150	......
....	......	....	......	10.22	0–1	10.33	0–1	*Ti* II	3409.82	4	......
....	......	12.01	1	....	......	....	......	....	....	......	......
....	......	....	......	....	......	13.24	1–0	*Fe* I	3413.14	15	......
3414.15	0–1	4.21	0–1	3.83	0–1	....	......	*Fe* II	3414.14	2	......
3416.08	4	....	......	5.91	2	5.73	2	*Fe* II	3416.02	5	......
3418.06	0–1	....	......	....	......	....	......	(*Fe* II	3418.03	pr)	......
3419.33	0–1	....	......	....	......	9.71	1–0	(P II (*Fe* II	3419.25 3420.18	5) 0)	....
3421.26	7	1.02	2	1.19	5	1.23	3	*Cr* II	3421.20	75	......
3422.80	7	2.80	3	2.82	5	2.71	3	*Cr* II	3422.74	125	......
....	......	....	......	....	......	4.39	1–0	(*Fe* II	3424.17	pr)	......

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3425.52	2	....	......	5.41	1	....	......	*Fe* II	3425.58	3	......
3428.38	1 n	....	......	....	......	....	......	*Fe* II *Cr* II	3428.63 3428.94	pr 6	....
3430.70	0–1	....	......	....	......	30.36	1–0	(*Fe* II (*Cr* II	3430.15 3430.42	pr) 2)	....
3433.36	8	3.40	3	3.33	4	3.30	3	*Cr* II	3433.30	75	......
3436.09	3	....	......	5.90	1	6.38	2	*Fe* II	3436.11	5	......
3438.86	3	9.09	1–0	9.11	1	8.91	1 n	*Mn* II	3438.96	3	......
3440.63	1	0.96	1	0.41	1–0	0.72	1–0	*Fe* I	3440.63	150	......
3442.07	8	2.17	3	2.10	5	2.25	2	*Mn* II *Fe* II	3441.98 3442.24	30 3	+ ......
3444.23	4	4.32	3	4.09	2 n	3.84	1	*Ti* II	3444.33	30	......
3446.40	2	6.53	1	6.41	1 n	6.38	1–0	(*Ni* I *Co* II (*K* I	3446.27 3446.40 3446.72	100) 100 8)	....
3448.84	1	....	......	....	......	9.39	1	(*Fe* II	3448.43	1)	......
3450.98	0–1	....	......	....	......	1.22	2	*Cr* II *Fe* II *Fe* II	3450.84 3451.23 3451.32	3 2 2	....
3452.24	1	....	......	....	......	....	......	*Ti* II	3452.48	4	......
....	......	....	......	....	......	3.12	1 n	*V* II	3453.09	90	......
3454.09	2	....	......	3.91	1 n	....	......	*Fe* II *Ni* II	3453.60 3454.17	2 5	...... +
3454.72	1	....	......	....	......	....	......	*Cr* II	3454.98	25	......
3456.68	3	6.40	1 n	6.64	1 n	6.90	1 n	*Ti* II *Fe* II	3456.40 3456.93	20 5	....
3457.54	2	....	......	....	......	....	......	*V* II *Cr* II	3457.15 3457.62	300 25	...... +
....	......	8.47	0–1	....	......	9.31	1	*Cr* II	3459.29	18	......
3460.30	6	0.31	3	0.30	4	0.38	2	*Mn* II	3460.33	20	......
3461.51	4	1.57	2	1.58	2	1.74	1	*Ti* II	3461.50	20	......
3464.27	1 n	4.06	0–1	4.10	1	4.13	2	*Fe* II *Cr* II *Fe* II	3463.97 3464.02 3464.50	1 2 3	.... +
3465.68	3	5.82	1	5.75	2	....	......	*Ti* II *Ni* II	3465.65 3465.65	3 1	+ ......
....	......	....	......	....	......	6.86	1–0	*Cr* II	3467.14	......	......
3468.64	4	....	......	8.64	3	8.67	1	*Fe* II	3468.68	8	......
3471.48‖	4	....	......	1.23‖	2	0.77	1–0	*Fe* II *Ni* II *Cr* II	3470.24 3471.35 3472.07	1 2 15	...... + ...

‖ Blended in α Cygni but resolved in η Leonis.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
....	......	....	......	2.14	1	2.17	1	*Cr* II	3472.07	15	......
3474.06	6	3.95	4	4.11	4	4.04	4	*Fe* II	3473.82	2	......
								Mn II	3474.06	8	+
								Mn II	3474.15	8	+
3475.47	2	5.53	2	5.57	1	5.29	3	*Cr* II	3475.13	10	......
								Fe II	3475.25	pr	......
								Fe II	3475.74	pr	......
3477.16	4	6.97	2	7.05	2	7.17	1–0	*Ti* II	3476.99	tr	......
								Ti II	3477.19	15	+
3478.62	1–0	....	......	....	......	8.34	0–1	*Cr* II	3478.17	2	......
3479.78	1	....	......	9.79	1	....	......	*V* II	3479.84	80	......
								Fe II	3479.91	2	+
3482.85	6	2.87	2	2.83	4	2.53	2	*Fe* II	3482.43	2	......
								Cr II	3482.60	6	......
								Mn II	3482.91	12	+
3484.11	1	....	......	4.54	0–1	4.07	1–0	*Cr* II	3484.15	10	+
								Fe II	3484.35	1	......
3485.98	1–0	5.74	1	6.43	0–1	5.84	0–1	*V* II	3485.92	250	......
3488.66	5	8.69	2	8.62	3	8.68	2	*Mn* II	3488.68	10	......
3491.04	3	0.90	1 n	1.05	1	0.85	1	*Fe* I	3490.60	100	......
								Ti II	3491.06	10	+
3493.48	8	3.27	1	3.38	3	3.47	1	*V* II	3493.16	150	......
								Fe II	3493.47	10	+
3494.76	1	....	......	....	......	....	......	*Fe* II	3494.67	5	......
3495.70	8	5.55	2	5.82	3	5.60	4	*Cr* II	3495.37	15	......
								Fe II	3495.62	4	......
								Mn II	3495.84	7	+
3497.46	6 nv	7.74	2	7.65	2	7.77	3	*Mn* II	3496.81	3	......
								V II	3497.03	200	......
								Mn II	3497.53	6	+
								Fe II	3497.72	pr	......
3499.82	2	9.78	1	3500.02	1	00.04	1	*Fe* II	3499.88	4	......
3501.78	2	....	......	1.78	1	1.91	1	*Co* II	3501.73	200	......
3503.32	1–0	....	......	....	......	3.60	1	*Cr* II	3503.38	3	......
								Fe II	3503.47	2	......
3504.86	5	4.82	3	4.72	2	4.59	2	*V* II	3504.43	400	......
								Ti II	{3504.88, 3504.92}	80	+
3507.74	2 n	7.89	0–1	7.45	1 n	7.81	2 n	*Fe* II	3507.39	3	+
								Fe II	3508.21	1	......
3509.54	0–1	....	......	....	......	....	......	*Ti* II	3509.85	3	......
3510.79	4	0.86	1	0.57	2	0.58	1–0	*Ti* II	3510.85	60	......
3511.76	3	....	......	1.83	1	2.01	1	*Cr* II	3511.84	30	......
3514.01	5	3.90	1 n	3.97	3	3.74	1–0	*Cr* II	3513.05	10	......
								Fe I	3513.83	30	......
								Ni II	3513.94	8	+

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3517.13	2	......	......	7.89	1–0 n	......	......	*V* II	3517.30	800	......
3520.09	3	20.19	1	20.48	1–0	19.99	1	*V* II *Ti* II	3520.02 3520.26	120 20	...... +
......	......	......	......	......	......	2.15	0–1	*V* II *Fe* II *Cr* II	3521.84 3522.11 3522.15	90 pr 7	.... +
3524.47	1 n	4.29	1	......	......	......	......	*Ni* I *V* II	3524.54 3524.71	200 200	....
3526.52	1	......	......	5.98	1	6.30	1	*Fe* I	3526.04	20	......
3528.96	1–0	9.12	0–1	......	......	......	......	(*Cr* II	3528.26	1)	......
3530.84	2	......	......	......	......	......	......	*V* II	3530.76	500	......
3533.18	1	......	......	2.86	0–1	......	......	*Fe* I	3533.20	10	......
3535.48	3	5.42	1 n	5.34	1	5.61	2	*Ti* II *Fe* II	3535.41 3535.63	40 2	+ ......
3538.00	1	......	......	......	......	8.67	1–0	*Fe* II *V* II	3537.50 3538.24	1 50	....
3541.34	1 n	......	......	1.10	1–0	......	......	*Fe* I *V* II	3541.10 3541.34	15 50	....
3545.12	3	......	......	......	......	......	......	*V* II	3545.19	1000	......
......	......	......	......	9.11	1 n	......	......	*Fe* II	3549.03	1	......
3554.46	1	4.86	1–0	......	......	......	......	(*Lu* II *Fe* I *Fe* II	3554.45 3554.94 3555.08	200) 40 pr	...... + ...
3556.82	3	6.62	1	6.61	1 n	......	......	*V* II	3556.80	1500	......
3558.29	2	8.51	1	......	......	......	......	*Sc* II *Fe* I	3558.53 3558.53	20 30	....
3561.20	2 n	......	......	0.60	1–0 n	1.18	1 n	*V* II *Ti* II	3560.59 3561.58	90 3	...... +
3564.65	2	5.08	1	4.96	1–0 n	4.44	1	*Fe* II *Fe* I	3564.52 3565.40	pr 60	+ ......
3566.04	3	......	......	......	......	6.18	2	*Ti* II *Fe* II *Fe* II *V* II	3565.97 3566.05 3566.16 3566.18	6 2 3 200	+ ...... + ...
3567.68	1	......	......	......	......	8.55	1–0	*Sc* II	$\left\{\begin{matrix}3567.70\\3567.74\end{matrix}\right\}$	20	......
3570.01	2	0.03	3	0.13	1	0.06	2	*Fe* I	3570.14	100	......
3572.33	4	2.55	1	2.59	0–1	......	......	*Sc* II	$\left\{\begin{matrix}3572.48\\3572.57\end{matrix}\right\}$	50	......
3573.88	2	......	......	......	......	......	......	*Ti* II	3573.74	20	......
3576.67	7	6.30	1	6.57	2	......	......	*Sc* II *Ni* II	$\left\{\begin{matrix}3576.33\\3576.39\end{matrix}\right\}$ 3576.76	30 3	...... +

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3578.54	1–0 n	8.58	1	......	......	......	......	*Cr* I *Ti* II	3578.69 3578.71	200 0	....
3581.08	5	1.24	3	1.37	1 n	1.40	1 n	*Sc* II *Fe* I	3580.93 3581.21	20 250	...... +
3583.35	1–0	3.50	1	......	......	......	......	......	......	......	......
3585.36	10	5.27	3	5.35	5	5.53	6	*Cr* II *Cr* II	3585.31 3585.54	60 40	+ ......
3587.18	2	7.14	1–0	7.74	1	7.31	1	*Ti* II	3587.15	12	......
3589.72	3	9.89	1	9.55	2	......	......	*V* II	3589.74	1000	......
3592.12	2	......	......	......	......	......	......	*V* II	3592.01	800	......
3593.62	1	3.34	1	......	......	......	......	*V* II *Cr* I	3593.32 3593.50	600 160	+ ......
3595.95	3	6.07	1	5.79	1	......	......	*Ti* II	3596.06	50	......
3600.77	0–1	......	......	......	......	......	......	*Y* II	3600.74	300	......
3603.70	6	3.74	1	3.62	3	3.85	5	*Cr* II *Cr* II *Cr* II	3603.64 3603.80 3603.86	15 30 10	...... + ...
3605.93	1 n	......	......	5.72	1	5.47	1	*Cr* I *Fe* I	3605.34 3605.48	140 15	...... +
3608.72	3 n	8.73	1	8.91	2	8.97	1	*Cr* II *Fe* I	3608.66 3608.87	3 100	....
3610.60	1	0.40	1–0	......	......	......	......	*Y* II	3611.05	200	......
3613.44	4	3.56	1 n	......	......	3.11	1	*Cr* II *Ti* II *Sc* II	3613.21 3613.33 {3613.81 3613.88}	20 pr 100	+ ...... +
3614.84	2	......	......	4.76	1	4.96	1	*Fe* II	3614.87	5	......
3618.80	3 n	8.69	2	......	......	......	......	*Fe* I *V* II	3618.78 3618.92	125 200	....
3621.26	5	......	......	1.18	2	1.44	2	*V* II *Fe* II	3621.20 3621.27	150 6	...... +
3623.24	1	......	......	3.32	1	......	......	(*Zr* I	3623.92	300)	......
3624.84	4	5.08	1	4.56	2	5.02	2	*Ti* II	3624.84	70	+
3627.14	1	......	......	......	......	......	......	*Fe* II	3627.17	1	......
3628.68	1	8.11	0–1	......	......	8.22	1–0	*Y* II	3628.71	100	......
3631.48	8	1.41	2	1.32	3	1.74	4	*Fe* I *Cr* II *Cr* II	3631.47 3631.49 3631.72	125 50 25	...... + ...
3634.98	2 n	......	......	......	......	4.39	1–0	*Cr* II *Fe* II	3634.04 3634.89	5 5	...... +
3638.03	1	......	......	......	......	......	......	*Fe* I	3638.30	12	......
3641.31	4	1.12	1	0.92	1	1.50	1–0	*Fe* II *Ti* II	3641.22 3641.34	pr 100	...... +

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	I	λ	I	λ	I	λ	I	Element	λ	I	Contributor
3642.99	2	3.12	1	......	......	......	......	*Sc* II *Cr* II	{3642.78, 3642.83} 3643.22	50 8	....
3645.07	2	......	......	......	......	4.53	0–1	*Cr* II *Sc* II	3644.70 {3645.29, 3645.34}	8 15	....
3647.76	2	7.55	1 n	......	......	7.95	1	*Cr* II *Fe* I	3647.40 3647.85	5 100	....
......	......	9.38	1–0	......	......	......	......	*Fe* I	3649.51	12	......
3650.40	1	......	......	......	......	0.53	1	*Cr* II	3650.37	20	......
3651.70	1	1.64	1	......	......	......	......	*Cr* II	3651.68	8	......
3657.47	0–1	......	......	......	......	8.51	1	*Cr* II	3658.19	12	......
3659.64	2	9.38	0–1	......	......	60.50	0–1	*Ti* II	3659.76	60	......
3662.08	2	......	......	......	......	2.78	1	*V* II *Ti* II H_{30}	3661.38 3662.24 3662.26	200 40	.... +
3663.53	1	......	......	......	......	......	......	H_{29}	3663.40	......	......
3664.68	2	......	......	......	......	5.21¶	1	H_{28} *Cr* II	3664.68 3664.95	 20	+ ......
3665.98	2	......	......	6.29	1	......	......	H_{27}	3666.10	......	......
3667.74	2	......	......	8.24	1	......	......	H_{26}	3667.68	......	......
3669.45	4	......	......	9.45	2	......	......	*V* II H_{25}	3669.41 3669.47	300	...... +
3671.36	4	......	......	1.41	3	......	......	H_{24}	3671.48	......	......
3673.78	6	......	......	3.87	3	......	......	H_{23}	3673.76	......	......
3676.38	5	......	......	6.44	4	......	......	H_{22}	3676.36	......	......
3677.84	4	7.62	2	8.03	3	......	......	*Cr* II *Cr* II *Cr* II	3677.69 3677.86 3677.93	20 30 10	+ + ...
3679.43	6	9.57	1	9.46	6	......	......	H_{21}	3679.36	......	......
3682.78	8	2.77	3 n	2.85	8	......	......	H_{20}	3682.81	......	......
3685.22	5	5.13	5	5.28	5	5.05	2 n	*Ti* II	3685.20	250	......
3686.87	9	6.83	4 n	6.88	10	7.06	3 n	H_{19}	3686.83	......	......
3691.59	10	1.42	5 n	1.59	15	2.15	3 n	H_{18}	3691.56	......	......
3693.97	0–1	3.85	1	......	......	......	......	*Fe* I (*Yb* II	3694.03 3694.20	20 200)	....
3697.18	12	6.97	6	7.28	16	7.36	4 n	H_{17}	3697.15	......	......
3700.17	1	......	......	......	......	......	......	*V* II	3700.34	200	......
3703.78	15	3.74	5 n	3.93	18	3.93	5 n	H_{16}	3703.86	......	......
3706.07	6	5.90	3	6.38	5	......	......	*Ca* II *Ti* II	3706.04 3706.22	10 20	+ ......

¶ Probably *Cr* II in α Canum Venaticorum.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3709.93	2 n	......	......	......	......	9.78	2 n	*Fe* I *V* II *Y* II	3709.26 3709.34 3710.29	75 40 500	....
3711.97	15	1.99	7 n	2.07	20 n	2.30**	7 n	H_{15}	3711.97	.....	......
3712.88	5	......	......	......	......	......	......	*Cr* II	3712.97	40	......
3715.29	5	......	......	5.32	6	5.38	3	*Cr* II *Cr* II *V* II	3715.19 3715.45 3715.48	20 18 1200	+ + ...
......	......	......	......	......	......	8.16	1	*V* II	3718.16	60	......
3719.99	3	19.82	2	20.31	3	......	......	*Fe* I *Fe* II	3719.95 3720.17	250 pr	+ ......
3722.01	15	1.81	8 n	2.03	20 n	1.65	8 n	H_{14}	3721.94	.....	......
3723.69	0–1	......	......	......	......	......	......	*Cr* II *Ti* II *Fe* II *Ti* II	3723.40 3723.61 3723.91 3724.09	7 tr pr 1	+ ....
3725.40	2	......	......	......	......	5.03	1	*Fe* II	3725.30	3	......
3727.33	4	7.64	3 n	7.14	4	7.35	3	*V* II *Cr* II *Fe* I	3727.35 3727.37 3727.64	1000 10 50	+ + ...
3732.93	1	......	......	......	......	......	......	*V* II *Fe* I	3732.76 3733.33	800 40	....
3734.35	15	4.47	15 n	4.48	25 n	4.52	10 n	H_{13} *Fe* I	3734.37 3734.88	 300	+ ......
3736.96	6	6.95	6	7.14	7	......	......	*Ca* II *Fe* I	3736.92 3737.14	11 150	+ ......
3738.42	3	9.03	1	8.42	2	......	......	*Cr* II	3738.38	15	......
3741.69	6	1.48	2	1.67	5	1.82	3	*Ti* II	3741.65	50	......
3743.43	2	3.57	1	3.31	2	3.66	3	*Fe* I *V* II	3743.37 3743.61	20 40	....
3745.63	4	5.69	3	5.94	3	5.92	3	*Fe* II *Fe* I *V* II	3745.36 3745.58 3745.81	pr 100 800	....
3746.59	3	......	......	......	......	......	......	(*Fe* II	3746.56	pr)	......
3748.53	6	......	......	8.68	3	......	......	(*Ti* II (*Fe* I (*Cr* II	3748.00 3748.27 3748.68	10) 60) 5)	....
3750.17	15	0.23	20 n	0.12	20 n	0.20	12 n	H_{12} *V* II	3750.15 3750.88	 600	+ ......
3754.56	4	......	......	4.71	1 nn	......	......	*Cr* II	3754.59	12	......
3755.55	1	......	......	......	......	5.22	2	*Fe* II	3755.56	4	......
3757.79	5	8.28	2	8.00	2	8.04	1	*Ti* II *Fe* I	3757.69 3758.25	30 150	+ ......

** Perhaps blended with *Cr* II 3712.97 in α Canum Venaticorum.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3759.33	6	9.31	3	9.52	7	9.48	3	*Ti* II *Fe* II	3759.30 3759.46	200 6	+ ......
3761.45	6	1.43	4	1.44	5	1.77	3	*Ti* II *Cr* II *Ti* II *Cr* II	3761.32 3761.69 3761.88 3761.90	200 4 15 4	+ ....
3762.99	0–1	....	......	3.23	1	....	......	*Fe* II	3762.89	5	......
3763.98	4	3.59††	2	4.01	4	3.91	3	*Fe* I *Fe* II	3763.80 3764.11	100 pr	....
3765.55	2	....	......	5.69	1	5.73	1	*Cr* II	3765.62	8	......
3767.33	1	....	......	....	......	....	......	*Cr* II *Fe* I	3766.65 3767.21	2 80	...... +
3769.63	5	....	......	....	......	....	......	*Ni* II	3769.46	5	......
3770.77	7	0.57	20 n	0.77	25 n	0.68	15 n	H_{11} *V* II	3770.63 3770.97	 400	+ ......
3774.11	1	....	......	....	......	....	......	*V* II *Ti* II	3774.34 3774.65	300 2	....
3776.03	2	5.68	1	....	......	....	......	*Ti* II	3776.06	6	......
3778.17	1	....	......	....	......	....	......	*V* II	3778.36	100	......
3779.53	2	....	......	9.08	1 n	9.05	2 n	*Fe* II	3779.57	pr	......
3781.53	1	....	......	1.51	1	1.97	1	*Fe* II	3781.51	1	......
3783.42	5	....	......	3.36	3	3.53	1	*Fe* II	3783.35	4	......
3786.14	1	....	......	....	......	....	......	(*Ti* II	3786.33	pr)	......
....	......	....	......	....	......	7.02	1	*V* II	3787.24	150	......
3788.11	1	7.56	1–0	....	......	8.35	1	*V* II *Fe* I	3787.24 3787.89	150 50	....
3794.98	1	....	......	....	......	....	......	*V* II *Fe* I	3794.37 3795.01	50 60	...... +
3797.97	20	7.89	20 n	8.07	30 n	7.91	20 n	H_{10}	3797.90	.....	......
....	......	....	......	....	......	805.71	0–1	*Fe* I	3805.35	12	......
3806.63	2 nn	....	......	6.46	1 n	6.88	1	*Fe* I	3806.70	10	......
3810.02	1–0 n	....	......	....	......	....	......	......	....	.....	......
3813.40	1	3.17	1	....	......	....	......	*Fe* I *Ti* II	3812.96 3813.40	40 2	....
3814.23	4	....	......	4.19	3	4.23	3	*Fe* II *Ti* II	3814.12 3814.60	4 4	+ ......
3815.84	3	5.97	2	5.81	1	6.27	1	*V* II *Fe* I	3815.38 3815.85	200 100	...... +
3820.35	4	0.33	3	19.91	4 n	20.41	1	*Fe* I	3820.44	250	......
3821.85	3	....	......	1.76	1	2.10	1	*Fe* II	3821.97	pr	......

†† Mostly *Fe* I in α Lyrae.

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3824.86	4	4.45	2	4.90	3	5.17	2	*Fe* II	3824.91	4	......
3825.75	3	6.11	3	....	......	....	......	*Fe* I	3825.89	200	......
3827.43	3	7.69	2	7.23	2	7.56	2	*Fe* II *Fe* II *Fe* I	3827.08 3827.69 3827.83	4 pr 75	....
3829.45	3	9.33	3	9.34	2	....	......	*Mg* I	3829.37	40	......
3832.40	4	2.10	4	2.21	3	....	......	*Mg* I	3832.31	80	......
3835.37	20	5.35	20 n	5.38	20 n	5.27	20 n	H_9	3835.39	......	......
3838.27	5	8.20	4	8.02	4	....	......	*Fe* II *Mg* I	3838.04 3838.30	2 100	...... +
3840.94	3	1.00	2 n	0.63	2	1.13	1	*Fe* I *Fe* I *Fe* II	3840.45 3841.06 3841.36	80 80 pr	+ + ...
3843.24	1	....	......	2.94	1–0	....	......	*Mn* II	3842.98	1	......
3845.29	3	5.53	1–0	4.93	2	5.26	2	......	....	......	......
3847.77	1	....	......	8.15	1	....	......	*V* II *Mg* II	3847.32 3848.24	100 7	....
3849.73	3	9.74	1 n	9.85	3	50.24	1 n	*Fe* I	3849.98	40	......
3851.46	0–1	....	......	....	......	....	......	(*Fe* I	3850.83	12)	......
3853.74	3	3.11	1 n	3.86	4	3.65	3	*Si* II	3853.67	3	......
3856.05	5	6.23	4	6.09	6	6.10	6	*Si* II *Fe* I	3856.03 3856.38	8 50	+ ......
3860.00	3	59.85	3	59.78	1	....	......	*Fe* I *Fe* II	3859.92 3860.11	300 pr	+ ......
3862.61	6	2.70	3	2.62	5	2.60	3	*Si* II	3862.60	6	......
3864.03	0–1	....	......	....	......	....	......	*Fe* II *V* II *Fe* II	3863.41 3863.81 3863.94	1 60 1	....
3865.44	2	5.80	2	5.51	2	5.52	2	*Fe* I *Cr* II	3865.54 3865.59	30 20	...... +
3867.75	1–0	....	......	7.16	1	8.19	1–0	......	....	......	......
....	......	9.15	0–1	9.68	1–0	....	......	......	....	......	......
3872.68	4	2.38	2	2.66	2 n	2.59	1	*Fe* I	3872.51	60	......
3875.70	1 n	4.64	1–0	5.73	0–1 n	....	......	......	....	......	......
3878.56	4 n	8.43	2	8.65	1 n	9.18	2	*Fe* I *V* II	3878.58 3878.72	100 300	....
3880.94	0–1	....	......	....	......	....	......	*Fe* II	3880.78	1	......
3882.48	2	2.31	1 n	....	......	....	......	(*Mn*	3883.28	3)	......
3886.61	1	....	......	....	......	....	......	*Fe* I	3886.30	40	......
3889.06	20	9.30	25 n	8.90	20 n	9.12	18 n	H_8	3889.05	......	......

TABLE 1—*Continued*

α Cygni		α Lyrae		η Leonis		α Can. Ven.		Identification			
λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*	Contributor
3895.96	2	......	......	......	......	......	......	*Fe* I *V* II	3895.67 3896.16	25 60	....
......	......	......	......	......	......	8.46	1–0	*V* II	3899.14	200	......
3900.56	5	0.70	3	0.51	3	0.50	1	*Ti* II	3900.54	70	......
3903.36	2	3.22	1	3.37	1 n	3.46	1	*V* II	3903.27	250	......
3905.80	4	5.78	2	5.74	3	6.05	3	*Cr* II *Fe* II	3905.64 3906.04	18 5	....
3908.36	1–0	......	......	7.88	1	8.39	1	......	......	......	......
3910.99	1–0	10.03	1–0	10.45	1 n	......	......	......	......	......	......
3913.50	5	3.52	3	3.54	3	......	......	*Ti* II	3913.47	60	......
3914.26	3	......	......	......	......	4.07	2 n	*V* II *Fe* II	3914.33 3914.48	250 2	....
3916.36	2	......	......	6.64	0–1	......	......	*V* II	3916.42	200	......
3918.43	1	......	......	9.11	1–0	8.54	1	(*Mn* II	3918.32	3)	......
3920.32	1	......	......	0.70	1	0.74	1	*Fe* I	3920.27	20	......
3922.88	1	3.16	1–0	......	......	......	......	*Fe* I	3922.92	25	......
3926.38	1	......	......	6.48	1 n	......	......	*Mn* II	3926.46	4	......
3927.95	1	......	......	......	......	......	......	*Fe* I	3927.94	30	......
3930.21	3	......	......	0.25	1	0.24	2	*Fe* I	3930.31	25	......
3932.15	2	......	......	......	......	......	......	*Ti* II	3932.02	2	......
3933.64	15	......	......	......	......	......	......	*Ca* II	3933.68	400	......
3936.01	3	......	......	......	......	......	......	*Fe* II	3935.94	6	......
3938.58	5 nr	......	......	......	......	......	......	*Fe* II *Fe* II	3938.29 3938.97	2 4	....
3941.70	1–0 n	......	......	......	......	......	......	......	......	......	......
3945.04	2 n	......	......	......	......	......	......	*Fe* II	3945.23	pr	......
3947.92	1	......	......	......	......	......	......	*O* I *O* I *O* I	3947.33 3947.51 3947.61	10 7 4	....
3952.03	2	......	......	......	......	......	......	*V* II	3951.97	500	......
3956.74	2	......	......	......	......	......	......	*Fe* I	3956.69	12	......

NOTES TO TABLE 1

Ti II The limiting laboratory intensity recorded in α Cygni is about 2. The following table gives an approximate conversion from laboratory intensities to stellar intensities.

Lab.	*	Lab.	*
2	0–1	30	4–5
5	1	45	4–5
6	2	70	4–5
10	2	100	5
12	2	125	6
20	2–3	200	6

Fe II The limiting laboratory intensity is 2 or 3.

Lab.	*	Lab.	*
1	0	7	3
2	0–1	8	3–4
3	1	9	5
4	2	10	7
5	2–3		

Cr II The limiting laboratory intensity is about 4.

Lab.	*	Lab.	*
2	0	12	2–3
3	0	20	2–3
4	0–1	35	3
5	1	50	5
6	1–2	75	5
7	1–2	150	6
8	2		

V II The limiting laboratory intensity is about 90.

Lab.	*	Lab.	*
90	0–1	300	2
100	1	500	2
200	2	800	3

Mg I

Lab.	*
40	3
80	4

Si II The limiting laboratory intensity is about 2.

Lab.	*
2	0
3	3
8	5

P II Very improbable. Only λ 3419.25 may be present; there are no other coincidences.

K I Very improbable because $IP = 4.33$ v. It is possible that λ 3446.72 is present.

Sc II

Lab.	*
20	1
50	3

Cr I

Lab.	*
200	1–0

Mn II

Lab.	*
1	1
3	3
11	5–6
30	8

Fe I The limiting laboratory intensity is about 10.

Lab.	*	Lab.	*
8	0	100	1–2
20	0–1	200	3
30	1	250	4

Co II The strongest laboratory line λ 3501.73 is probably present. The ionization potential is 17.1 volts.

Ni I

Lab.	*
150	0
200	1

Ni II The limiting laboratory intensity is about 3.

Lab.	*
3	0–1
5	1–2
8	4

Y II The limiting laboratory intensity is about 100.

Lab.	*
100	0–1
200	0–1
300	1
500	2

Zr II Uncertain. Only one line present.
La II Absent in this region.
Yb II One tentative coincidence. Improbable.
Lu II Improbable. Lab. 200; *1

There remain very few unidentified lines. The strongest is λ 3845.29, intensity 3 in α Cygni.

TABLE 2

B STARS

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	*I*	λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*
3261.12	3	...	...	...	...	...	...	...	...	*O* III	3260.98	8
3263.40	3	...	...	...	...	...	...	...	...	(*Ne* II	3263.44	3)
3265.43	3	...	...	...	...	...	...	...	...	*O* III	3265.46	10
3267.38	1	...	...	...	...	...	...	...	...	*O* III (*Fe* III	3267.31 3266.88	5 20)
...	...	...	...	...	...	79.06	1–0	...	...	...	...	...
3281.40	0–1	...	...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	96.51	1–0	...	...	*He* I	3296.76	1
...	...	97.83	3 n	...	...	...	...	...	...	*Ne* II	3297.74	8
3309.63	0–1	...	...	...	...	...	...	...	...	...	...	...
3312.62	2	...	...	...	...	...	...	...	...	*O* III	3312.30	5
...	...	...	...	...	...	19.62	1	...	...	...	...	...
3321.37	1–0	...	...	...	...	...	...	...	...	...	...	...
3323.94	1	3.70	2	3.95	1	...	...	...	...	*S* III *S* III	3324.00 3324.85	4 6
3327.15	1	7.09	1	...	...	...	...	...	...	*Ne* II	3327.22	5
...	...	9.13	1	...	...	...	...	...	...	*N* II *Ne* II	3328.80 3329.18	4 4
3330.41	1	...	...	...	...	...	...	...	...	(*N* II *O* III	3330.30 3330.40	2) 4
...	...	1.39	1–0	...	...	...	...	...	...	*N* II	3331.32	3
3332.44	1–0	...	...	...	...	...	...	...	...	*O* III	3333.00	4
3334.92	3	4.75	4	4.77	2	...	...	...	...	*Ne* II	3334.87	10
...	...	6.29	1–0	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	39.14	1	...	...	*Fe* III	3339.36	10
3340.66	4	0.50	1	...	...	...	...	...	...	*O* III	3340.74	6
3341.86	2	1.83	1–0 n	...	...	...	...	...	...	...	...	...
3344.66	1 n	4.27	2 n	4.24	1	...	...	...	...	*Ne* II	3344.44	5
...	...	5.59	1	...	...	...	...	...	...	...	...	...
3347.66	1	7.81	1–0	...	...	...	...	...	...	(*Fe* III	3347.70	8)
...	...	9.57	1 n	...	...	...	...	...	...	...	...	...
3351.00	2	...	...	...	...	...	...	...	...	*O* III *O* III	3350.68 3350.99	3 4
3354.70	2 nn	4.73	2 n	4.58	2	4.39	2	4.82	3	*He* I	3354.52	1
...	...	6.45	0–1	...	...	...	...	...	...	*Ne* II	3355.09	7
3358.17	1–0	8.32	1 n	8.87	1–0	...	...	...	...	(*F* III	3358.32	4)

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	I	λ	I	λ	I	λ	I	λ	I	Element	λ	I
3360.68	1	0.55	2	0.61	1–0	61.04	0.1	...	...	*Ne* II	3360.63	5
...	...	3.21	1–0 n	...	...	...	...	...	...	(*O* III	3362.38	4)
3366.10	1	5.85	0–1 n	...	...	...	...	...	...	...	...	...
3367.42	3	7.25	2	7.08	1	...	...	...	...	*Ne* II *N* III	3367.25 3367.36	6 7
...	...	71.51	1 n	...	...	...	...	...	...	(*S* III	3370.37	4
3372.52	1	...	...	...	...	...	...	...	...	*Ca* III	3372.68	8
3374.14	1	...	...	...	...	...	...	...	...	*N* III	3374.06	6
...	...	7.03	1	...	...	...	...	...	...	*O* II	3377.20	7
3383.81	0–1	...	...	...	...	...	...	3.79	4	(*O* III *Ti* II (*O* III (*O* III	3382.69 3383.76* 3383.85 3384.95	3) ... 2) 4)
...	...	...	...	7.31	1	...	...	...	...	*S* III	3387.12	4
...	...	8.37	1	...	...	...	...	...	...	...	...	...
...	...	90.11	1–0	90.10	2	...	...	...	...	*O* II	3390.25	8
3392.44	1–0	2.91	1–0	2.88	1–0	...	...	...	...	...	...	...
...	...	...	...	...	...	4.95	1–0	...	...	...	...	...
...	...	3407.13	1	07.31	1	...	...	...	...	*O* II *N* II	3407.38 3408.14	7 3
...	...	09.95	0–1	...	...	...	...	...	...	*O* II	3409.84	6
...	...	...	...	...	...	...	...	412.53	1–0	...	...	...
...	...	...	...	413.70	1–0	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	414.80	1–0	...	...	...
3417.69	1	7.45	1 n	7.95	1	...	...	...	...	...	...	...
3419.92	0–1	...	...	...	...	9.22	1–0 n	...	...	...	...	...
3429.28	0–1	...	...	...	...	9.90†	1	...	...	*O* III	3428.67	3
3430.84	1	...	...	...	...	...	...	...	...	*O* III	3430.60	4
...	...	...	...	...	...	...	...	32.35	1–0	...	...	...
...	...	...	...	6.97	2	...	...	7.30	2	*N* II	3437.16	6
3440.10	0–1	...	...	...	...	...	...	...	...	*O* III	3440.39	4
3444.13	2	4.31	1	...	...	...	...	...	...	*O* III	3444.10	5
3447.53	2	7.55	3	7.59	3	7.59	3	7.37	3	*He* I	3447.59	2
3451.03	1–0	...	...	...	...	...	...	...	...	*O* III (*B* II	3450.94 3451.25	4 10)
3455.02	1–0	...	...	...	...	...	...	...	...	*O* III *O* III	3454.90 3455.12	2 5

* Interstellar.

† The line in γ Pegasi cannot be *O* III.

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	*I*	λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*
...	...	6.94	1–0	...	...	...	...	...	...	(*He* I	3456.7	1)
...	...	...	...	...	...	64.96	1–0	...	...	...	...	...
...	...	...	...	...	...	67.40‡	1–0	66.75‡	1 nn	*He* I	3465.89	1
...	...	70.43	2	...	...	...	...	...	...	*O* II *O* II	3470.42 3470.81	5 8
								71.84	1 nn	*He* I	3471.78	...
...	...	75.45	0–1	...	...	...	...	...	...	...	...	...
...	...	77.72	0–1	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	78.61	1 nn	*He* I	3478.95	1
...	...	80.52	1–0	...	...	...	...	...	...	...	...	...
...	...	81.88	1	81.94	1–0	...	...	...	...	...	...	...
...	...	84.63	0–1	...	...	...	...	...	...	...	...	...
...	...	87.09	1	7.03	1	7.46	1 nn	7.48	1 nn	(*Si* III *He* I	3486.93 3487.72	6) 1
...	...	...	...	...	...	...	...	93.00	1–0 nn	...	...	...
...	...	97.23	3	7.38	2	7.42	1 n	7.25	0–1 n	*S* III	3497.34	5
...	...	...	...	...	...	...	...	98.48	1 n	*He* I	3498.63	1
...	...	...	...	99.22	1–0	...	...	...	...	...	...	...
...	...	...	...	...	...	3501.40	1 n	...	...	(*F* II	3501.40	10)
...	...	3503.69	1 n	...	...	...	...	...	...	(*F* II (*F* II	3502.95 3503.10	8) 12)
...	...	...	...	...	...	06.12	1	...	...	(*F* II	3505.61	15)
...	...	...	...	...	...	...	...	09.46	1	...	...	...
...	...	10.97	2	...	...	...	...	...	...	...	...	...
...	...	...	...	11.82	1 nn	11.95	1 nn	12.32	2 nn	*He* I	3512.50	1
...	...	...	...	...	...	...	...	17.15	0–1	...	...	...
...	...	...	...	...	...	23.84	1–0	...	...	...	...	...
3530.45	1 nn	30.51	1 nn	0.07	1 nn	0.37	2 n	0.34	3	*He* I	3530.50	1
...	...	...	...	...	...	...	...	32.35	0–1	...	...	...
3537.20	1–0 nn	...	...	...	...	...	...	...	...	*Ca* III	3537.75	7
3542.91	1–0 n	1.72	1 nn	...	...	2.30	2 n	...	...	*Ne* II	3542.89	7
3554.10	1 nn	3.93	1 nn	...	...	4.29	3 n	4.60	4	*He* I	{3554.44 3554.57}	1
...	...	7.40	0–1 n	...	...	...	...	...	...	...	...	...
...	...	...	...	63.58	1–0 n	...	...	...	...	...	...	...
3568.49	2	8.48	3	...	...	...	...	...	...	...	...	...

‡ The *He* line is present in 55 Cygni but is not present in γ Pegasi, because of Stark effect.

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	I	λ	I	λ	I	λ	I	λ	I	Element	λ	I
....	......	70.92	0–1	......	......	......	......	......	......	......	......	...
3571.75	0–1	......	......	......	......	......	......	......	......	......	......	...
3574.99	0–1	4.52	2	......	......	......	......	......	......	......	......	...
....	......	81.46	1–0 n	......	......	......	......	......	......	......	......	...
....	......	......	......	82.92	1–0 n	......	......	......	......	......	......	...
3587.04	2 nn	7.05	3 nn	7.11	3 n	7.33	3	7.42	5	*C* II *He* I *C* II	3585.83 {3587.28 3587.42} 3587.68	2 2 2
....	......	90.36	2	0.56	2 n	0.31	1	0.51	0–1	*C* II *Si* III *C* II	3589.67 3590.46 3590.87	4 8 2
....	......	94.10	1 n	4.36	1	......	......	......	......	*N* II	3593.60	3
....	......	......	......	......	......	......	......	96.37	0–1	......	......	...
....	......	......	......	99.30	1–0	99.40	1	......	......	*He* I	{3599.32 3599.46}	1
....	......	......	......	3601.22	1	01.74	2	01.36	1	(*Fe* III *Al* III	3600.93 3601.62	10) 6
....	......	......	......	......	......	......	......	04.98	0–1	(*Fe* III	3603.88	9)
....	......	......	......	......	......	......	......	07.95	0–1	......	......	...
3609.17	1 n	9.68	1	9.74	1	......	......	......	......	*C* III *C* III	3608.96 3609.40	4 5
....	......	......	......	12.45	0–1	12.37	1	12.57	1	*Al* III	3612.35	4
3613.54	2	3.63	4	3.60	3	3.68	4	3.74	3	*He* I	3613.64	3
3618.65	0–1	......	......	......	......	......	......	......	......	......	......	...
....	......	20.20	1–0	......	......	......	......	......	......	......	......	...
....	......	......	......	32.03	1	......	......	......	......	*S* III	3632.03	6
....	......	......	......	......	......	32.94	1–0	......	......	......	......	...
3634.27	4 n	4.36	4 n	4.16	7	4.39	6	4.24	6	*He* I	{3634.24 3634.37}	2
....	......	44.19	1	......	......	......	......	......	......	......	......	...
....	......	......	......	......	......	49.99	0–1	......	......	......	......	...
....	......	52.22	1	2.12	1	2.13	1	2.07	2	*He* I	{3652.00 3652.12}	1
....	......	62.10	1–0	......	......	......	......	......	......	*S* III	3662.00	5
3663.88	1	4.08	2	......	......	......	......	......	......	*Ne* II	3664.09	9
....	......	......	......	......	......	......	......	69.46	0–1	H_{25}	3669.47	....
....	......	......	......	......	......	......	......	71.66	1 S	H_{24}	3671.48	....
....	......	......	......	......	......	......	......	74.13	1 S	H_{23}	3673.76	....
....	......	......	......	......	......	......	......	76.62	2 n	H_{22}	3676.36	....

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	I	λ	I	λ	I	λ	I	λ	I	Element	λ	I
....	......	79.86	0–1 nn	......	......	....	......	79.60	3 n	H_{21}	3679.36	....
....	......	....	......	......	......	....	......	82.72	4 n	H_{20}	3682.81	....
....	......	....	......	......	......	86.10	1 nn	86.60	4 n	H_{19}	3686.83	....
....	......	....	......	91.31	1 n	92.03	1 nn	91.64	5 n	H_{18}	3691.56	....
3694.33	2	4.19	4	4.30	1	....	......	....	......	*Ne* II	3694.22	10
3696.76	1	6.77	1	7.52	1 n	6.67	2 nn	6.97	5	(*O* III H_{17}	3695.37 3697.15	5) ....
3699.15	1	....	......	......	......	....	......	....	......	*O* III	3698.70	5
3703.65	2	....	......	......	......	3.40	1–0	3.91	4	(*O* III H_{16}	3703.37 3703.86	5) ....
3705.30	5	4.93	5 n	5.01	10 n	4.99	10	4.98	4	*He* I	{3705.00 3705.14}	3
3707.12	1	7.36	1–0	......	......	....	......	....	......	*O* III	3707.24	6
3709.75	2	9.75	3	9.97	2	....	......	....	......	*Ne* II	3709.64	7
....	......	....	......	......	......	12.04	4 nn	11.97	6	H_{15}	3711.97	....
3712.88	3	3.40	4	2.69	5	....	......	....	......	*O* II *Ne* II	3712.75 3713.09	7 10
3714.85	2	....	......	......	......	....	......	....	......	*O* III	3715.08	6
3718.12	1	7.77	1	......	......	....	......	7.91	1	*S* III	3717.77	7
3722.07	3 nn	1.68	2 nn	1.87	4 n	1.95	5 nn	1.87	10	H_{14}	3721.94	....
3725.50	1–0	....	......	......	......	....	......	5.48	1	......	....	....
3727.35	2	7.23	4	7.21	4	7.55	1	7.19	2	*Ne* II *O* II	3727.08 3727.33	9 8
....	......	33.18	2	......	......	....	......	....	......	*He* I	{3732.85 3732.99}	1
3734.27	3 nn	....	......	4.09	10 n	4.12	10 nn	4.22	15	H_{13}	3734.37	....
....	......	5.34	2 n	......	......	....	......	....	......	*Ne* II (*O* II	3734.94 3735.94	7 3)
....	......	41.60	1 n	......	......	....	......	....	......	*O* II	3739.92	6
3748.08	1–0 n	....	......	......	......	....	......	....	......	*S* III	3747.90	5
3750.25	4 nn	49.92	5 n	49.88	12 n	50.12	15 nn	50.25	15	(*O* II H_{12}	3749.49 3750.15	9) ....
....	......	....	......	......	......	....	......	52.46	1–0	......	....	....
3754.71	4	4.69	2	......	......	....	......	....	......	*N* III *O* III	3754.62 3754.67	6 7
....	......	....	......	......	......	....	......	55.87	1	*He* I	3756.09	1
3757.26	3	7.34	1	......	......	....	......	....	......	*O* III	3757.21	5
3759.96	5	60.00	2	......	......	....	......	....	......	*O* III	3759.87	9
3762.36	2	2.45	1	2.52	1	....	......	....	......	*O* II *Si* IV	3762.63 3763.41	5 4

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	*I*	λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*
3766.46	1–0 nn	6.53	2	5.92	1–0	......	......	......	......	*Ne* II	3766.29	8
3770.65	5 nn	0.55	10 n	0.51	15 n	0.65	15 nn	0.67	18	(*He* I (*He* I H_{11}	3768.80 3770.57 3770.63	1) 1) ...
3773.83	2	3.88	1–0	......	......	......	......	......	......	(*Si* IV *O* III	3773.13 3774.00	3) 6
3776.98	1	7.30	2	7.05	1	......	......	6.66	1–0	*Ne* II (*O* II	3777.16 3777.60	8 4)
......	......	......	......	......	......	85.26	1–0	84.85	3	*He* I	3784.88	1
......	......	87.57	1–0	......	......	......	......	......	......	*He* I	3787.49	1
......	......	......	......	89.21	1–0	......	......	......	......	......	......	...
3791.41	3	1.45	2	1.53	2	......	......	1.87	1	*O* III *Si* III	3791.26 3791.41	6 3
3797.93	7 nn	8.05	10 nn	7.90	20 n	7.90	15 nn	8.01	18	H_{10}	3797.90	...
......	......	......	......	3802.87	1	......	......	......	......	*O* II	3803.14	6
......	......	3806.55	4	06.65	6	06.60	3	06.46	5	(*He* I *Si* III	3805.75 3806.56	1) 5
3819.77	10	9.78	7 n	9.75	10	9.74	15	9.65	12	*He* I	{3819.61 3819.75}	4
......	......	......	......	29.78	1	......	......	......	......	*N* II	3829.80	3
3835.45	10 n	5.82	15 nn	5.42	20 n	5.35	20 nn	5.41	18	(*He* I H_9	3833.56 3835.39	1) ...
......	......	......	......	8.45	3	......	......	8.41	3	*He* I *S* III *N* II	3838.09 3838.32 3838.39	1 8 5
......	......	......	......	43.01	1	......	......	......	......	*N* II	3842.20	3
......	......	......	......	47.47	1	......	......	......	......	*N* II	3847.38	3
3849.12	1–0	......	......	......	......	......	......	......	......	......	......	...
......	......	......	......	51.13	1	51.01	1–0	50.56	......	(*Cl* II (*O* II (*Cl* II	3850.99 3851.04 3851.37	25) 3) 30)
......	......	......	......	56.47	1	55.94	1	56.03	3	(*N* II *N* II *O* II	3855.08 3856.07 3856.16	2) 3 5
......	......	......	......	60.94	1	......	......	60.39	0–1	*S* III (*Cl* II (*Cl* II	3860.64 3860.83 3860.98	4 35) 15)
......	......	......	......	......	......	62.44	1–0	62.81	1	(*Si* II	3862.60	6)
......	......	64.53	1	4.27	2	4.72	0–1	......	......	*O* II	3864.45	5
3867.42	2	7.76	3	7.59	3	7.66	3	7.50	4	*He* I	{3867.46 3867.62}	2
......	......	71.67	1 n	71.72	2 n	72.07	2 n	71.68	4	*He* I	3871.80	1
......	......	76.39	2	6.20	3	6.34	2	......	......	*C* II *C* II	3876.19 3876.41	4 2

TABLE 2—*Continued*

10 Lacertae		τ Scorpii		β Cephei		γ Pegasi		55 Cygni		Identification		
λ	*I*	λ	*I*	λ	*I*	λ	*I*	λ	*I*	Element	λ	*I*
....	.....	82.34	1	2.13	3	2.31	1	2.97	1–0	*O* II	3882.19	7
3888.89	15 n	9.12	15 nn	8.79	20 n	8.95	20 n	9.16	20	*He* I H_8	3888.64 3889.05	10 ....
....	.....	....	.....	3906.26	0–1 n	....	.....	....	.....	.....	....	....
....	.....	3907.34	1	....	.....	....	.....	....	.....	*O* II	3907.45	4
....	.....	12.21	2	12.27	3	11.81	1	11.72	1 n	*O* II	3911.95	10
....	.....	....	.....	....	.....	....	.....	16.31	1–0	.....	....	....
3919.87§	1 n	9.37	2	9.29	3	8.93	3	9.14	3	*C* II *N* II *O* II	3918.98 3919.00 3919.28	6 6 6
....	.....	....	.....	20.37	1	20.77	3	20.64	3	*C* II	3920.68	8
....	.....	24.87	1–0 n	4.89	2	4.88	2	4.42	1	*Si* III	3924.44	4
3926.74	1 nn	7.31	2 n	6.93	3 n	6.94	3 n	6.67	4	*He* I	3926.53	1
3934.08‖	4	3.79‖	1 n	....	.....	....	.....	....	.....	*Ca* II (*N* III	3933.66 3934.41	.... 3)
3938.82	1	....	.....	....	.....	....	.....	....	.....	*N* III	3938.52	4
....	.....	44.95	1	....	.....	....	.....	....	.....	*O* II	3945.05	5
....	.....	49.68	1–0	....	.....	....	.....	....	.....	.....	....	....
....	.....	54.55	2	....	.....	....	.....	....	.....	*O* II *N* II	3954.37 3955.85	7 6
3961.60	2	1.79	1	....	.....	....	.....	....	.....	(*S* III *O* III	3361.55 3361.59	4) 8
3964.98	3	4.83	5	....	.....	....	.....	....	.....	*He* I	3964.73	4
3968.22¶	2	....	.....	....	.....	....	.....	....	.....	*Ca* II	3968.46	....
3970.43	10 n	....	.....	....	.....	....	.....	....	.....	$H\epsilon$	3970.08	....

§ In 10 Lacertae blended with *C* II 3920.68.

‖ Probably interstellar.

¶ Interstellar.

NOTES TO TABLE 2

He I Many new *He* I lines have been found in this region. The members of the two diffuse series extend very far in the supergiant 55 Cygni. The highest members become appreciably broad and diffuse—indicating the presence of Stark effect even in these stars. No Stark broadening is recorded in P Cygni (Struve and Roach, *Ap. J.*, **91**, 727, 1939).

B II Only one very weak line in 10 Lacertae. Improbable.

C II Present.

C III Definitely present.

N II Present.

N III Present.

O II Present.

O III Strong in 10 Lacertae.

F II Probably present.

F III One line probably present.

Ne II Strong and numerous.

Mg II Absent in this region.

Si II Probably absent in this region.

Si III Present.

Si IV Present.

P III Absent in this region.

S II Absent in this region.

S III Present.

Fe III A few weak lines present. Very strong in P Cygni.

Cl II Uncertain. λ 3860.8 may be blended with *S* III 3860.64 and λ 3851 may be blended with *O* II 3851.04. These are the strongest laboratory lines.

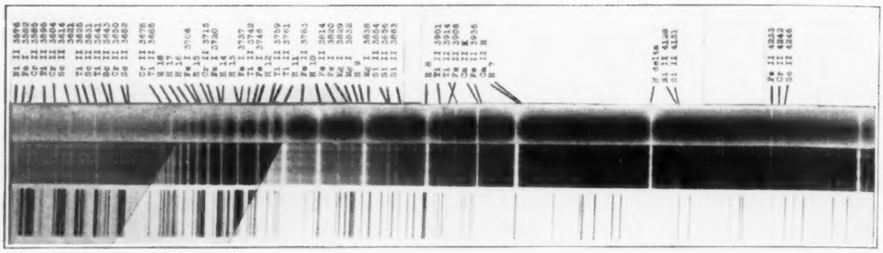
Ar II The strongest line in this region, λ 3545.84, is absent.

Ca III Suggested by coincidences of two strongest laboratory lines λ 3372.68 (8) and λ 3537.75 (7) in 10 Lacertae.

Several fairly strong star lines remain unidentified.

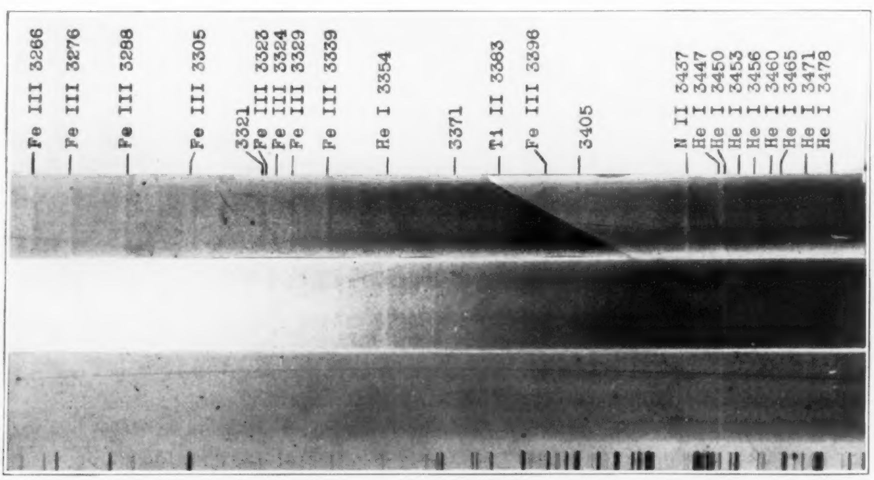
YERKES OBSERVATORY
August 1939

PLATE XVII



a

Spectra of 17 Leporis (*top*) and α Cygni (*bottom*)



b

Spectra of P Cygni (*top*), τ Scorpii (*middle*), and 55 Cygni (*bottom*)

THE ULTRAVIOLET SPECTRA OF 17 LEPORIS AND P CYGNI*

O. STRUVE AND F. E. ROACH

ABSTRACT

The ultraviolet spectrum of 17 Leporis is compared with that of α Cygni. *Ti* II, *Sc* II, and *Mn* II are greatly enhanced in 17 Leporis. *Si* II is abnormally weak. From the relative intensities of three *Mg* I lines it is found that the dilution factor $W > 10^{-4}$. Other estimates suggest $0.1 > W > 0.01$. The observations are, in general, consistent with the theory of cycles.

One hundred and fifteen lines were measured in the spectrum of P Cygni between λ 3265 and λ 4026. The Balmer series is followed to $n = 23$. The triplet diffuse series of *He* I is found to extend to appreciably higher members than the singlet diffuse series. The 2^3P level is estimated to be overpopulated by a factor of 3. This could be caused by a dilution factor $W = 0.05$, which corresponds to $R/r = 0.4$, where R and r are the radii of the star and of the shell, respectively. The *Si* III triplet, $4p^3P^o - 4d^3D$, is found to be in emission in contrast with the well-known triplet $4s^3S - 4p^3P^o$, which is in absorption. Several strong lines are due to *Fe* III. The effect of dilution in this spectrum is discussed.

17 LEPORIS

The spectrum of 17 Leporis presents a striking case of departures from thermodynamic equilibrium, presumably produced by dilution of the exciting stellar radiation in an extended atmosphere.[1] The ordinary photographic spectrum has been described previously.[2] A spectrogram obtained with the Cassegrain spectrograph of the McDonald Observatory on March 5, 1939, covers the region λ 3300–*Hα*. A reproduction of the violet part of this spectrogram is shown in Plate XVII*a*. *Hα* is a strong bright line having absorption borders on both sides. This confirms the existence of a large nebulous shell around the star—which had already been inferred from a study of the absorption lines.

Table 1 gives a list of those lines which are appreciably strengthened in 17 Leporis, as compared with α Cygni. The wave lengths in the first column are approximate and have been taken from the list of lines of α Cygni by Wyse,[3] whenever they were given in that paper. A few faint lines have been measured separately and com-

* *Contributions from the McDonald Observatory, University of Texas*, No. 14.

[1] Struve, *Proc. Amer. Phil. Soc.*, **81**, 241, 1939.

[2] Struve, *Ap. J.*, **76**, 85, 1932.

[3] *Lick Obs. Bull.*, No. 492, 1938.

TABLE 1

LINES WHICH ARE ENHANCED IN 17 LEPORIS

λ	Most Important Element	Lower E.P.	Int. Sun	Multiplet
3361.3	*Ti* II	0.028	3	$a^4F_{3\frac{1}{2}}-z^4G^{\circ}_{4\frac{1}{2}}$
3372.7	*Ti* II	0.012	5	$a^4F_{2\frac{1}{2}}-z^4G^{\circ}_{3\frac{1}{2}}$
3379.9	*Ti* II	0.049	3	$a^4F_{4\frac{1}{2}}-z^4G^{\circ}_{4\frac{1}{2}}$
3383.8	*Ti* II	0.000	3	$a^4F_{1\frac{1}{2}}-z^4G^{\circ}_{2\frac{1}{2}}$
3387.8	*Ti* II	0.028	5d?	$a^4F_{3\frac{1}{2}}-z^4G^{\circ}_{3\frac{1}{2}}$
3391.5	*Cr* II	2.411	2	$a^4D_{\frac{1}{2}}-z^4P^{\circ}_{1\frac{1}{2}}$
3394.3	*Ti* II	0.012	3	$a^4F_{2\frac{1}{2}}-z^4G^{\circ}_{2\frac{1}{2}}$
3403.4	*Cr* II	2.424	2	$a^4D_{1\frac{1}{2}}-z^4P^{\circ}_{1\frac{1}{2}}$
3421.2	*Cr* II	2.411	4	$a^4D_{\frac{1}{2}}-z^4P^{\circ}_{\frac{1}{2}}$
3442.0	*Mn* II	1.768	6	$a^5D_4-z^5P^{\circ}_3$
3444.3	*Ti* II	0.150	4	$b^4F_{4\frac{1}{2}}-z^4G^{\circ}_{5\frac{1}{2}}$
3456.6	*Ti* II	2.052	3	$b^2P_{1\frac{1}{2}}-y^2P^{\circ}_{1\frac{1}{2}}$
3460.3	*Mn* II	1.802	4d?	$a^5D_3-z^5P^{\circ}_2$
3461.6	*Ti* II	0.134	5	$b^4F_{3\frac{1}{2}}-z^4G^{\circ}_{4\frac{1}{2}}$
3474.1	*Mn* II	1.802	2	$a^5D_3-z^5P^{\circ}_3$
3477.2	*Ti* II	0.121	5	$b^4F_{2\frac{1}{2}}-z^4G^{\circ}_{3\frac{1}{2}}$
3482.9	*Mn* II	1.825	5d?	$a^5D_2-z^5P^{\circ}_2$
3488.7	*Mn* II	1.840	4	$a^5D_1-z^5P^{\circ}_1$
3491.1	*Ti* II	0.112	5	$b^4F_{1\frac{1}{2}}-z^4G^{\circ}_{2\frac{1}{2}}$
3495.8	*Mn* II	1.847	2	$a^5D_0-z^5P^{\circ}_1$
3499.8	....	....	....	....
3503.3	....	....	....	....
3504.9	*Ti* II	1.884	3	$b^2G_{4\frac{1}{2}}-y^2G^{\circ}_{4\frac{1}{2}}$
3510.7	*Ti* II	1.885	5	$b^2G_{3\frac{1}{2}}-y^2G^{\circ}_{3\frac{1}{2}}$
3517.1	*V* II	1.123	3	$a^3F_4-z^5D^{\circ}_3$
3520.1	*Ti* II	2.039	2	$b^2P_{\frac{1}{2}}-x^2D^{\circ}_{1\frac{1}{2}}$
3533.2	....	....	....	....
3556.9	*V* II	1.123	4	$a^3F_4-z^3D^{\circ}_3$
3572.7*	*Sc* II	0.022	4–6	$a^3D_3-z^3D^{\circ}_3$
3581.2	*Fe* I	0.855	30	$a^5F_5-z^5G^{\circ}_6$
3589.5	*V* II	1.067	5d?	$a^3F_2-z^5F^{\circ}_1$
3596.0	*Ti* II	0.605	4	$a^2F_{3\frac{1}{2}}-z^4D^{\circ}_{2\frac{1}{2}}$
3613.7†	*Cr* II	2.695	1N	$a^4P_{\frac{1}{2}}-z^6D^{\circ}_{\frac{1}{2}}$
	Sc II	0.022	4	$a^3D_3-z^3F^{\circ}_4$
3618.8	*Fe* I	0.986	20	$a^5F_2-z^5G^{\circ}_3$
3624.6	*Ti* II	1.216	5	$a^2P_{\frac{1}{2}}-z^2S^{\circ}_{\frac{1}{2}}$
3630.8	*Sc* II	0.008	4	$a^3D_2-z^3F^{\circ}_3$
3641.2	*Ti* II	1.232	4	$a^2P_{1\frac{1}{2}}-z^2S^{\circ}_{\frac{1}{2}}$
3642.9‡	*Sc* II	0.000	3	$a^3D_1-z^3F^{\circ}_2$
3645.0	....	....	....	....

* Very strong.

† Mostly due to *Sc* II in 17 Leporis.

‡ Mostly *Sc* II in 17 Leporis but may be *Cr* II in α Cygni.

TABLE 1—*Continued*

λ	Most Important Element	Lower E.P.	Int. Sun	Multiplet
3651.7	*Sc* II	0.008	4	$a^3D_2-z^3F^\circ_2$
3655.3	....	....	....	....
3658.4	....	....	....	....
3659.7	*Ti* II	1.575	5	$b^2D_{2\frac{1}{2}}-y^2F^\circ_{3\frac{1}{2}}$
3662.0	*Ti* II	1.559	5	$b^2D_{1\frac{1}{2}}-y^2F^\circ_{2\frac{1}{2}}$
3677.9	*Cr* II	2.693	3	$a^4P_{2\frac{1}{2}}-z^4P^\circ_{1\frac{1}{2}}$
3685.2	*Ti* II	0.605	10d?	$a^2F_{3\frac{1}{2}}-z^2D^\circ_{2\frac{1}{2}}$
3705.6	*Fe* I	0.051	9	$a^5D_3-z^5F^\circ_3$
	Ca II	3.110	6d?	$4^2P_{\frac{1}{2}}-5^2S^\circ_{\frac{1}{2}}$
3715.3	*Cr* II	3.090	2	$b^4D_{3\frac{1}{2}}-z^4F^\circ_{4\frac{1}{2}}$
3720.2	*Fe* I	0.000	40	$a^5D_4-z^5F^\circ_5$
3737.0	*Fe* I	0.051	30	$a^5D_3-z^5F^\circ_4$
	Ca II	3.137	5	$4^2P_{1\frac{1}{2}}-5^2S^\circ_{\frac{1}{2}}$
3741.5	*Ti* II	1.575	4	$b^2D_{2\frac{1}{2}}-y^2D^\circ_{2\frac{1}{2}}$
3745.7	*Fe* I	0.087	8	$a^5D_2-z^5F^\circ_3$
3759.4	*Ti* II	0.605	12d?	$a^2F_{3\frac{1}{2}}-z^2F^\circ_{3\frac{1}{2}}$
3761.4	*Ti* II	0.571	7	$a^2F_{2\frac{1}{2}}-z^2F^\circ_{2\frac{1}{2}}$
3900.6	*Ti* II	1.126	5	$a^2G_{4\frac{1}{2}}-z^2G^\circ_{4\frac{1}{2}}$

pared with new measurements of α Cygni made on McDonald Observatory spectrograms. The identifications are also mostly those given by Wyse. The strongest contributing line has been listed in each case. In a few cases the relative importance of the blended components in 17 Leporis differs appreciably from that in α Cygni.

The most significant fact revealed by Table 1 is the great strengthening of *Ti* II. Apparently all multiplets are enhanced in 17 Leporis. All observable lines start from metastable levels, except $a^4F - z^4G^\circ$, which starts from the ground level. The lines of this multiplet are probably enhanced even more than those of other multiplets which start from excited metastable levels. In comparing the lines of 17 Leporis with those of α Cygni, it should be remembered that practically all lines of the former fall on the 45° branch of the curve of growth.[4] The curve of growth of α Cygni is not accurately known; but the turbulence is certainly less than in 17 Leporis, and only the weaker lines fall on the 45° branch.[5] Because of the uncertainty in the curve of growth of α Cygni, it is not possible to ascertain definite-

[4] Struve and Elvey, *Ap. J.*, **79**, 428, 1934. [5] *Ibid.*, p. 427.

ly whether the low-level lines are enhanced more than those of higher excitation potential; but there is a suspicion that such may be the case. The first two ionization potentials of *Ti* are 6.8 volts and 13.6 volts.

We next notice the remarkable enhancement of the lines of *Mn* II. All members of the multiplet $a^5D - z^5P^o$ are greatly enhanced in 17 Leporis. The lower level is metastable, and its excitation potential is 1.8 volts. The first two ionization potentials of *Mn* are 7.4 volts and 15.7 volts.

Even more striking is the great enhancement of the line of *Sc* II. All belong to the multiplets $a^3D - z^3D^o$ and $a^3D - z^3F$, both of which originate from the ground level. The line λ 4247 ($a^1D - z^1D^o$), the lower level of which is metastable, with an excitation potential of 0.3 volt, is also enhanced in 17 Leporis, although perhaps not quite as much as the lines of the multiplet $a^3D - z^3F^o$. The first two ionization potentials of *Sc* are 6.6 volts and 12.8 volts.

The lines of *Cr* II show a slight enhancement in 17 Leporis, but the effect is not very conspicuous. All lines originate from metastable levels. The first two ionization potentials are 6.7 volts and 16.6 volts. There are some indications that *V* II lines are enhanced and that *Ni* II is about as strong as in α Cygni. These lines start from metastable levels.

Of particular interest is *Fe* I. Several lines of the multiplet $a^5D - z^5F^o$, starting from the ground level, are greatly enhanced in 17 Leporis. Appreciable enhancement is also observed in the multiplet $a^5D - z^5D^o$, originating from the ground level, and in $a^5F - y^5F^o$, originating from a low metastable level of excitation potential 0.9 volt. The strongest lines originating from higher levels—for example, λ 4045 ($a^3F_4 - y^3F_4^o$), excitation potential 1.5 volts—are slightly weakened in 17 Leporis. But so are also the fainter members of the low-level multiplets—for example, λ 3927.935 ($a^5D_1 - z^5D_2^o$). Hence the effect is certainly in part produced by differences in the curve of growth. Whether there remains an excess in favor of the lowest multiplets is not certain.

The lines of *Mg* I 3838, 3832, and 3829 ($3^3P^o - 3^3D$) are of special interest in the study of departures from thermodynamic equilibrium.[6] The lower levels, $3^3P_0^o$ and $3P_2^o$, are metastable, while level

[6] Struve, *Proc. Amer. Phil. Soc.*, **81**, 235, 1939.

$3^3P^o_1$ combines with the ground level of the singlet system, giving the intersystem line λ 4571.104 ($3^1S_0 - 3^3P^o_1$). Hence, we should expect that the level $3^3P^o_1$ would be reduced in population when the radiation is diluted. The observational data for Nova Herculis were inconclusive,[6] but in 17 Leporis the three lines are clearly resolved from the hydrogen line H_9. There is no appreciable weakening of λ 3832. It is considerably weaker than λ 3838; but, on the other hand, it is stronger than λ 3829. Hence it is reasonable to attribute the slight differences from the relative intensities in α Cygni, where λ 3832 is nearly equal to λ 3838, to differences in the curves of growth. Evidently dilution does not sensibly affect the population of state $3^3P^o_1$.

If we consider three states—3^1S_0, $3^3P^o_1$, and a composite high level which includes the high discrete states—as well as the state of ionization, we have for the relative populations[7] of states $3^3P^o_1$ and 3^1S_0

$$\frac{n_2}{n_1} = W\rho_{12}\left[\frac{\gamma + 3 + a\rho_{23}}{\gamma + 3 + aW\rho_{23}}\right],$$

where

$$W = \text{the dilution factor } \frac{R^2}{4r^2},$$

$$\gamma = \frac{A_{31}}{A_{32}},$$

$$a = \frac{A_{31}}{A_{21}},$$

$$\rho_{12} = e^{\frac{-h\nu_{12}}{kT}},$$

$$\rho_{23} = e^{\frac{-h\nu_{23}}{kT}}.$$

The lifetime of the state $3^3P^o_1$ in *Mg* I is approximately 4×10^{-3} sec.[8] That of the combined upper state may be assumed to be of the order of 10^{-8} sec. Hence we have, roughly,

$$a = 4 \times 10^5 .$$

[7] *Proc. Amer. Phil. Soc.*, **81**, 221, 1939.

[8] A. C. J. Mitchell and W. M. Zemansky, *Resonance Radiation and Excited Atoms*, p. 147, New York, 1934.

Assuming for the difference in excitation potential of levels two and three, 3 volts, and $T = 15,000°$, we have

$$\rho_{23} = 0.1 .$$

In order that the population of level $3^3P_1°$ be essentially that given by the Boltzmann formula, it is necessary that

$$aW\rho_{23} \gg \gamma + 3 .$$

The value of γ is of the order of 1. Hence in our case

$$W > 10^{-4} .$$

This inequality is in good agreement with other estimates of W. From the absence of strong emission lines produced by the expanding shell we infer that the radius of the shell cannot be excessively large. In other, similar stars an estimate of $W = 0.01$ was deduced from the helium lines.[9]

It would be extremely interesting to investigate the intensities of the *Mg* I lines in shells for which $W < 10^{-4}$, but at the present time no such observations are available.

Table 2 lists those lines which are appreciably weaker in 17 Leporis than in α Cygni. Most conspicuous is the multiplet of *Si* II $sp^2\ {}^2D - 4^2P°$, λλ 3854, 3856, and 3863. The ground level is odd. Hence, the lower level of the multiplet is normal, and the theory fully accounts for the weakness of the lines. Very faint traces of all three lines are present, and a value of W between 0.1 and 0.01 seems fairly reasonable. The faintness of λ 4128 and λ 4131, $3^2D - 4^2F°$, has been pointed out previously. The ionization potentials of *Si* are 8.12 volts and 16.27 volts.

The only other lines in Table 2 are due to *Fe* II. All but one of these arise from normal lower levels (c^2F, d^2D) which are higher than the lowest odd levels ($z^6D°$, $z^4F°$, etc.). These lines should all be faint. The great majority of *Fe* II lines in the ordinary photographic region arises from metastable levels. The strongest of these lines, λ 4233, $b^4P_{2\frac{1}{2}} - z^4D°_{3\frac{1}{2}}$, is of about the same intensity in 17 Leporis

[9] Struve and Wurm, *Ap. J.*, **88**, 105, 1938.

and in α Cygni; but the fainter lines are generally weaker in 17 Leporis. The one anomalous line in Table 2, λ 3783.4, arises from a metastable level. The line is strong in α Cygni and, if the identification is correct, should be fairly strong in 17 Leporis. No satisfactory explanation has been found for this anomaly. The first two ionization potentials of *Fe* are 7.8 volts and 16.5 volts.

The extreme weakness of the ultimate lines of *Sr* II 4078 and 4216, $5^2S - 5^2P^o$, has already been commented upon.[10] These lines are moderately strong in α Cygni, and in spite of differences in the curve of growth, we should have expected to find them reasonably strong in 17 Leporis. The ionization potentials are 5.6 volts and 10.1 volts.

TABLE 2

LINES WEAKENED IN 17 LEPORIS

λ	Most Important Element	Lower E.P.	Int. Sun	Multiplet
3783.4*	*Fe* II	2.267	2	$a^2P_{1\frac{1}{2}}-z^4D_{2\frac{1}{2}}$
3853.8	*Si* II	6.827	−1	$sp^2\,{}^2D_{1\frac{1}{2}}-4^2P_{1\frac{1}{2}}$
3856.1	*Si* II	6.829	1	$sp^2\,{}^2D_{2\frac{1}{2}}-4^2P_{1\frac{1}{2}}$
3862.8	*Si* II	6.827	1	$sp^2\,{}^2D_{1\frac{1}{2}}-4^2P_{1\frac{1}{2}}$
3905.9	*Fe* II	5.547	−1	$c^2F_{2\frac{1}{2}}-x^2G_{3\frac{1}{2}}$
3936.0	*Fe* II	5.545	2	$c^2F_{3\frac{1}{2}}-x^2G_{4\frac{1}{2}}$
4002.3	*Fe* II	5.930	abs.	$d^2D_{2\frac{1}{2}}-x^2F_{3\frac{1}{2}}$
	Fe II	2.766	0	$b^4P_{\frac{1}{2}}-z^4P_{1\frac{1}{2}}$

* This may not be all due to *Fe* II.

The ultimate lines of *Ca* II are about equally strong in 17 Leporis and α Cygni. The latter also shows the lines λ 3706 and λ 3737, $4^2P^o - 5^2S$, but in 17 Leporis both are heavily blended with *Fe* I. This is unfortunate, since the *Ca* II lines would otherwise furnish an interesting test of the theory.

Table 3 summarizes the results. The first part of the table contains those elements which are strengthened in 17 Leporis as compared with α Cygni, or which are of comparable intensity in both stars (*Mg* I, *Fe* II metastable). The second part shows those elements which are abnormally weak in 17 Leporis. The great weakness of *Sr* II cannot be explained by the theory of cycles. It is prob-

[10] *Proc. Amer. Phil. Soc.*, **81**, 242, 1939.

ably due to the fact that the second ionization potential is low, so that most atoms of strontium are doubly ionized. This, however, does not seem compatible with the great strengthening of *Ti* II, *Sc* II, and *Fe* I and with the weakening of *Fe* II. However, the present theory is quite inadequate to explain the observational details. Perhaps the best procedure would be to determine the degree of ionization from the ultimate lines of *Ca* I (λ 4226) and *Ca* II (H and K). It will then be possible to study the relative abundances of those elements for which we observe the resonance lines (*Ti* II, *Sc* II, *Fe* I, and *Sr* II) and then attempt to infer whether the intensities of those lines of *Fe* II which start from metastable levels are normal. But this work must be postponed until an accurate curve of growth for α Cygni is available.

TABLE 3

STRONG LINES IN 17 LEPORIS

(In Order of Decreasing Ratio $\frac{\text{17 Leporis}}{\alpha \text{ Cygni}}$)

Element	*Ti* II	*Mn* II	*Sc* II	*Cr* II	*Ni* II	*V* II	*Fe* I	*Mg* I	*Fe* II
First I.P.	6.8	7.4	6.6	6.7	7.6	6.8	7.8	7.6	7.8
Second I.P.	13.6	15.7	12.8	16.6	18.2	14.7	16.5	15.0	16.5
Average E.P.	1	1.8	0–0.3	2–3	3	1	0–1	2.7	2–3

WEAK LINES IN 17 LEPORIS

(In Order of Decreasing Ratio $\frac{\text{17 Leporis}}{\alpha \text{ Cygni}}$)

Element	*Fe* II (Normal)	*Si* II	*Mg* II	*Sr* II
First I.P.	7.8	8.1	7.6	5.6
Second I.P.	16.5	16.3	15.0	11.0
Average E.P.	5–6	7–10	9	0

P CYGNI

An excellent spectrogram of this star on Process emulsion was obtained with the ultraviolet Cassegrain spectrograph of the McDonald Observatory, on May 26, 1939. The linear dispersion varies from 21.7 A/mm at λ 3300 to 52.9 A/mm at λ 4300. The star

TABLE 4

ULTRAVIOLET LINES IN P CYGNI

λ* (Meas.)	Abs. or Em.	Int.	Identification			R.V. (Corrected to Sun)
			Elem.	λ (Lab.)	Transition	
3265.47	A	8	*Fe* III	6.88	$a^5F_5-z^5G^\circ_6$	−113
3266.59	E	4				− 11
3274.72	A	7	*Fe* III	6.08	$a^5F_5-z^5G^\circ_5$	−108
3275.64	E	4				− 24
3287.32	A	8	*Fe* III	8.81	$a^5F_3-z^5G^\circ_4$	−120
3288.47	E	5				− 15
3303.42	A	8	*Fe* III	5.22	$a^5F_2-z^5G^\circ_3$	−147
3304.99	E	5				− 5
3321.98	E	2	....	....	....	....
3323.41	A	6	*Fe* III	4.82	Unclassified	−111
3324.86	E	2				+ 20
3329.30	E	2	*Fe* III	9.89	Unclassified	+ 70
3337.68	A	6	*Fe* III	9.36	$a^5F_1-z^5G^\circ_2$	....
3338.76	E	4	*Fe* III	8.72	Unclassified	....
3352.98	A	4n	*He* I	4.55	2^1S-7^1P	−122
3354.32	E	1				− 2
3371.11	A	1s	....	....	....	....
3383.47*	A	4s	*Ti* II	3.76	$a^4F_{1½}-z^4G^\circ_{2½}$	− 10
3396.48	E	2	*Fe* III	6.71	Unclassified	+ 36
3405.50	E	2	....	....	....	....
3435.96	A	4	*N* II	7.16	$3s^1P^\circ_1-3p^1S_0$	− 89
3445.97	A	4	*He* I	7.59	2^1S-6^1P	−125
3448.98	A	1	*He* I	0.22	2^3P-21^3D	−102
3451.62	A	2	*He* I	3.21	2^3P-20^3D	−122
3455.60	A	2	*He* I	6.79	2^3P-19^3D	− 87
3459.32	A	2	*He* I	0.94	2^3P-18^3D	−124
3463.60	A	2	*He* I	5.91	2^3P-17^3D	−184
3469.41	A	2	*He* I	1.80	2^3P-16^3D	−191

* Interstellar.

TABLE 4—*Continued*

λ* (Meas.)	Abs. or Em.	Int.	Identification: Elem.	Identification: λ (Lab.)	Identification: Transition	R.V. (Corrected to Sun)
3477.69	A	3	*He* I	8.97	2^3P-15^3D	−94
3486.23†	E	4	*Si* III	6.93	Unclassified	−44
3497.45	A	5	*He* I	8.64	2^3P-13^3D	−86
3499.28	E	1				+71
3500.21	A	3	*F* II	1.42	$3p^5P_1-3d^5D^\circ_0$	....
3501.48	E	3	*Fe* III	0.29 1.75	Unclassified Unclassified	....
3502.70	A	3	*F* II	2.95 3.10	$3p^5P_2-3d^5D^\circ_{2,3}$	−11
3505.46	A	3	*F* II	5.61	$3p^5P_3-3d^5D^\circ_4$	....
3506.37	E	3	*Fe* III	6.93	Unclassified	....
3511.25	A	6	*He* I	2.51	2^3P-12^3D	−92
3512.39	E	1				+6
3529.06	A	6	*He* I	0.49	2^3P-11^3D	−106
3530.38	E	2				+7
3553.07	A	5	*He* I	4.50	2^3P-10^3D	−105
3554.43	E	1				+10
3574.08	A	1	....	....	....	....
3584.51	A	7	*Fe* III	6.12	$a^3F_4-z^3F^\circ_4$	−119
3585.78	E	2				−12
3587.24‡	E	4	*He* I	7.33	2^3P-9^3D	+8
3589.79	E	6	*Si* III	0.46	$4p^1P_1-4d^1D_2$	−40
3599.18§	A	10	*Fe* III	0.93	$a^3F_3-z^3F^\circ_3$	−130
3600.83	E	8				+8
3602.28	A	10	*Fe* III	3.88	$a^3F_2-z^3F^\circ_2$	−117
3603.76	E	8				+6
3611.97	A	8	*He* I	3.64	2^1S-5^1P	−123
3613.65	E	6				+17
3632.90	A	8	*He* I	4.28	2^3P-8^3D	−98
3634.28	E	6				+16

† *He* I 3487.97 visible as absorption line on either side of the *Si* III emission line.

‡ Probably an absorption line to violet of this emission line.

§ *He* I 3599.39 may be blended with this line.

TABLE 4—*Continued*

λ* (Meas.)	Abs. or Em.	Int.	Identification			R.V. (Corrected to Sun)
			Elem.	λ (Lab.)	Transition	
3647.88	A	6				
3649.29	E	6	....	....	....	....
3650.97	A	4	*He* I	2.06	2^3P-8^3S	....
3652.68	E	3	*Fe* III	2.65	Unclassified	....
3672.09	A	4	*H* I	3.76	2–23	−121
3674.66	A	4	*H* I	6.36	2–22	−123
3677.65	A	6	*H* I	9.36	2–21	−124
3681.26	A	6	*H* I	2.81	2–20	−110
3685.02	A	8	*H* I	6.83	2–19	−131
3689.82	A	8	*H* I	1.55	2–18	−125
3695.00	A	8	*H* I	7.15	2–17	−158
3696.85	E	3				− 8
3699.37	A	1	....	....	....	....
3703.14	A	12	*He* I	5.07	2^3P-7^3D	....
3705.08	E	9	*H* I	3.86	2–16	....
3710.78	A	6	*H* I	1.97	2–15	−141
3712.26	E	7				+ 37
3720.22	A	11	*H* I	1.94	2–14	−123
3722.09	E	7				+ 28
3726.54	A	3	*O* II	7.33	....	....
			Ne II	7.08	....	....
			V II	7.49	....	....
3732.59	A	10	*H* I	4.37	2–13	−127
3734.38	E	9				+ 17
3744.89	A	1	....	....	....	....
3748.46	A	11	*H* I	0.15	2–12	−119
3750.14	E	9				+ 15
3766.59	A	1s	....	....	....	....
3768.86	A	11	*H* I	0.63	2–11	−125
3770.47	E	9				+ 3
3790.98	E	2	*Si* III	1.41	$4p^3P_0-4d^3D_1$	− 18

TABLE 4—*Continued*

λ* (Meas.)	Abs. or Em.	Int.	Identification: Elem.	Identification: λ (Lab.)	Identification: Transition	R.V. (Corrected to Sun)
3796.23	A	11	*H* I	7.90	2–10	−116
3797.80	E	10				+8
3806.12	E	6	*Si* III	6.56	$4p^3P_2-4d^3D_1$	−19
3817.96	A	11	*He* I	9.61	2^3P-6^3D	−114
3819.47	E	9				+5
3833.48	A	14	*H* I	5.39	2–9	−133
3835.28	E	12				+7
3836.96	A	4	*He* I	8.09	....	....
			N II	8.39	....	....
			S III	8.32	....	....
3858.74	A	5n	*S* III	0.64	....	....
			Cl II	0.91	....	....
3865.35	A	4n	*He* I	7.54	2^3P-6^3S	−154
3870.12	A	4n	*He* I	1.80	2^1P-9^1D	−114
3875.92	A	1	*He* I	8.18	2^1P-9^1S	−159
3881.10	A	1	*O* II	2.19	$e^4D_{3½}-z^4D_{3½}$	−76
3886.30	A	25	*He* I	8.64	2^3S-3^3P	....
3888.48	E	30	*H* I	9.03	2–8	....
3924.92	A	6	*He* I	6.53	2^1P-8^1D	−107
3933.41‖	A	8s	*Ca* II	3.67	$4^2S_{½}-4^2P_{1½}$	−4
3954.56	A	2s	*N* II	5.85	$3s^3P_1-3p^1P_1$	−82
3957.48	A	1	....	....	....	....
3962.58	A	12	*He* I	4.73	$2^1S_0-4^1P_1^\circ$	−147
3964.60	E	11				+6
3967.92	A	25	*H* I	0.08	2–7	−147
3970.02	E	25				+11
3993.59	A	9	*N* II	5.00	$3s^1P_1-3p^1P_1$	−90
3994.96	E	5				+13
4003.16	A	3	....	....	....	....
4007.67	A	7	*He* I	9.27	$2^1P_1^\circ-7^1D_2$	−104
4009.38	E	3				+8
4024.20	A	20	*He* I	6.19	2^3P-5^3D	−132
4025.85	E	15				−9

‖ Interstellar.

spectrum is measurable from λ 3265 to *Hβ*. Since the region from *Hβ* to about λ 3900 was studied in detail by Struve[11] on a plate of somewhat greater dispersion, the present measurements were limited

TABLE 5

He LINES IN P CYGNI

	n	Absorption Int.	Absorption Vel.	Emission Int.	Emission Vel.
2^1S-n^1P					
5015	3	7	−180	10	− 6
3964	4	12	−147	11	+ 6
3613	5	8	−123	6	+17
3447	6	4	−125	....	.....
3354	7	4n	−122	1	− 2
2^1P-n^1S					
5047	4	1	−176	....	.....
4437	5	2	−104	1	+16
4168	6	1	− 83	....	.....
4023	7	....	......	....	.....
3935	8	....	......	....	.....
3878	9	1	−159	....	.....
2^1P-n^1D					
4921	4	10	−154	10	+ 6
4387	5	10	−123	6	−12
4143	6	9	−115	2	− 8
4009	7	8	−120	2	− 6
3926	8	6	−107	1	−15
3871	9	4d	−114	....	.....
3833	10	Blend with *H*			
3805	11	Just visible. May be blend with *Si* III			
3784	12	This line is not present			
3768	13	Blend with *H*			
3756	14	Absent			
2^3S-n^3P					
10830	2	....	......	....	.....
3888	3	Blend with *H* 3889			
3187	4	....	......	....	.....
2^3P-n^3S					
4713	4	7	−136	10	− 7
4120	5	6	−114	5	− 3
3867	6	4n	−154	....	.....
3732	7	Masked by *H* 3734			
3652	8	4	......	3	.....
3599	9	Masked by *Fe* III 3600.95			
2^3P-n^3D					
5875	3	....	......	....	.....
4471	4	15	−158	20	− 8
4026	5	15	−153	20	− 4
3819	6	11	−114	9	+ 5
3705	7	Blend with *H* 3704			
3634	8	8	− 98	6	+16
3857	9	Masked		4	+ 8
3554	10	5	−105	1	+10
3530	11	6	−106	2	+ 7
3512	12	6	− 92	1	+ 6
3498	13	5	− 86	1	+71
3487	14	Masked by *Si* III			
3478	15	3	− 94	....	.....
3471	16	2	−191	....	.....
3465	17	2	−184	....	.....
3460	18	2	−124	....	.....
3456	19	2	− 87	....	.....
3453	20	2	−122	....	.....
3450	21	1	−102	....	.....

on the long wave-length side by *He* 4026. The results are assembled in Table 4.

Helium.—The lines of six *He* I series represented in P Cygni[12] are assembled in Table 5. A special search was made for the higher

[11] *Ap. J.*, **81**, 66, 1935.

[12] The data for the lines of longer wave lengths than λ 4026 are taken from Struve, *ibid.*

members of the two series, $2^1P - n^1D$ and $2^3P - n^3D$. The extension of the triplet series to $n = 21$ is a striking feature of the spectrum. The singlet series extends to $n \leqq 11$. These facts suggest that the 2^3P and the 2^1P levels have relative populations which are not in accordance with their Boltzmann factors or, stated in another way, that there is a departure from thermodynamic equilibrium. The two levels are separated by 2048 cm^{-1}, which means that the relative Boltzmann factor, $2^1P : 2^3P$, should be 1.2 : 1. For practical purposes we may assume the two levels should be populated in the ratios of their statistical weights, 3 : 1, if thermodynamic equilibrium holds. To test this point a comparison was made with the stars 55 Cygni and τ Scorpii. The results are shown in Table 6 and in Figure 1.

TABLE 6

LAST VISIBLE MEMBER OF THE DIFFUSE *He* I SERIES

	Singlet	Triplet
P Cygni	11	21
55 Cygni	14	17
τ Scorpii	9	11

In the latter we have plotted against serial number n, the quantity log X'_0, which has been used by Goldberg[13] and which is proportional to the oscillator strength, f. The broken lines for τ Scorpii and 55 Cygni connect the last members of the $2^1P - n^1D$ and the $2^3P - n^3D$ series. These lines are roughly parallel. Hence, if we start from the fact that in P Cygni the last singlet line is $n = 11$, we should, by analogy, expect that the last triplet would be $n = 13.5$. In reality we observe $n = 21$. The difference in the ordinates of the triplets $n = 13.5$ and $n = 21$ gives us $\Delta \log X'_0 = 0.58$, from which we infer that the ratio of the populations $2^3P/2^1P$ is four times larger than the value found for thermodynamic equilibrium.

This result is complicated to some extent by the fact that Goldberg has included in his expression for log X'_0 the factor $(\lambda'/\lambda)^3$, which allows for a gradual change in the general opacity of the

[13] *Ap. J.*, **89**, 630, 1939.

atmosphere with λ. It is doubtful whether this law should be used for P Cygni. It is better to assume that the expanding shell is essentially transparent outside the lines. Since we are comparing

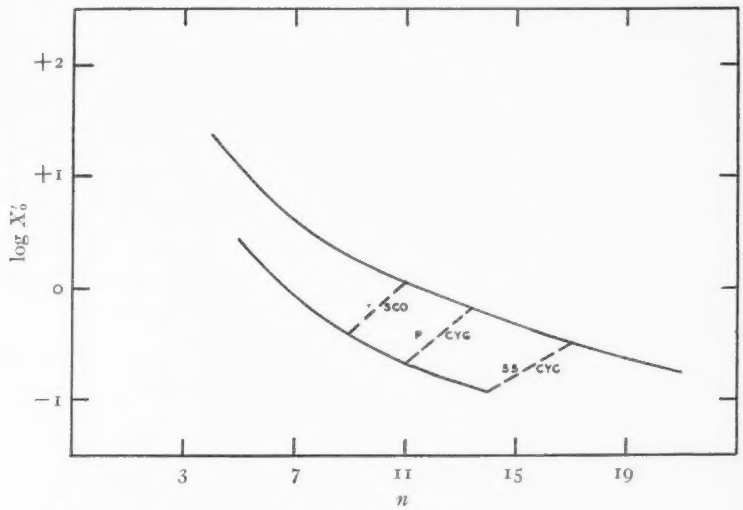


FIG. 1.—The upper curve represents the *He* I triplets; the lower curve, the *He* I singlets.

very weak lines, we shall suppose that the conditions for absorption in a thin slab are fulfilled. The equivalent widths of the last lines of the two series are

$$^1A_{11} = \frac{\pi e^2}{mc^2} \lambda_{11}^2 N_s H f_{(2-11)} ,$$

$$^3A_{21} = \frac{\pi e^2}{mc^2} \lambda_{21}^2 \, N_t H f_{(2-21)} .$$

From the observations it follows that

$$^1A_{11} = {}^3A_{21} .$$

Hence,

$$\lambda_{11}^2 N_s f_{(2-11)} = \lambda_{21}^2 N_t f_{(2-21)} ,$$

or

$$\frac{N_s}{N_t} = \left(\frac{3449}{3806}\right)^2 \frac{f_{(2-21)}}{f_{(2-11)}} = 0.82 \frac{f_{(2-21)}}{f_{(2-11)}} .$$

The values of f have been computed by Goldberg[14] only to $n = 10$. Hence, we adopt the asymptotic expressions:

$$f\ (\text{triplets}) = 4.69n^{-3}\ ,$$

$$f\ (\text{singlets}) = 3.33n^{-3}\ .$$

The result is $N_s/N_t = 0.17$, while the thermodynamic value is 0.3. The overpopulation of the 2^3P level corresponds to a factor of 2.

Under the circumstances the best that we can do is to take the mean resulting from the two methods used, giving an overpopulation by a factor of 3. This result shows fairly conclusively that the departure from thermodynamic equilibrium is present but that it is not excessively large. The computations by Struve and Wurm[15] show that for a temperature of 25,000° K the relative populations of the 2^3P and the 2^1P states vary with the dilution factor W as follows:

	$W=1$	$W=0.1$	$W=0.01$
$2^3P/2^1P$	3.1	7.1	13

Hence the best value for the dilution factor of the *He* shell in P Cygni is

$$W = 0.05\ .$$

This corresponds to a value of

$$\frac{R}{r} = 0.4\ ,$$

where R and r are the radii of the star and shell, respectively. This unexpectedly small extent of the expanding shell is in good agreement with other evidence.[16]

[14] *Ap. J.*, **90**, 414, 1939.

[15] *Ap. J.*, **88**, 98, 1938.

[16] Struve, *Proc. Amer. Phil. Soc.*, **81**, 242, 1939.

The stronger absorption lines of *He* I cannot be used for the evaluation of N_t and N_s because we have no knowledge of the curve of growth. In Figure 2 we have plotted the observed values of $\log A/\lambda$ against Goldberg's values of $\log X_0'$. The systematic difference between triplets (open circles) and singlets (closed circles) is not consistent with a single curve of growth for both series. The presence of strong emission lines in the triplets may account, in part, for this phenomenon. An inspection of the plate suggests that the singlet

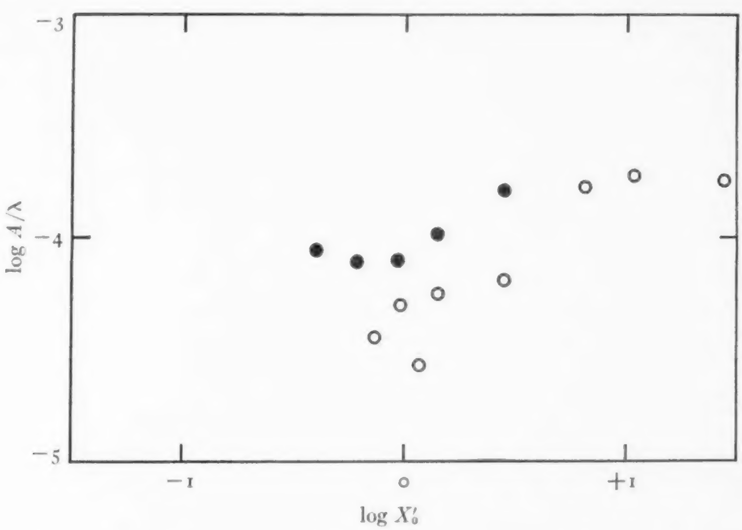


FIG. 2.—The diffuse triplets of *He* I are shown as open circles, the diffuse singlets as closed circles.

absorption lines are systematically broader than the triplets. The diagram is directly comparable to those published by Goldberg for various stars. He did not observe the ultraviolet triplets, and it should be of interest to investigate whether normal B stars show the same anomaly.

We have measured the equivalent widths of the *He* I absorption lines and the total emissions, referred to the energy included in one angstrom unit of the continuous spectrum, in the immediate vicinity

of the line. These values are given in the third and fourth columns of Table 7.

TABLE 7

He I LINES IN P CYGNI

Transition	λ	E.W.	Emission Intensity	$(1/W)N_nH$ ($\times 10^{-13}$)	$\frac{(1/W)N_nH}{ge^{-\chi/kT}}$ ($\times 10^{-18}$)
2^3P-4^3D	4471	0.815	1.275	2.3	1.4
5^3D	4026	.750	0.645	2.9	2.1
6^3D	3819	.645	0.245	2.2	1.8
8^3D	3634	.240	0.13	3.1	2.7
10^3D	3554	.200	....	....	....
11^3D	3530	.096	0.035	2.4	2.2
12^3D	3512	.161	0.063	5.8	5.4
13^3D	3498	.126	....	....	....
2^1P-5^1D	4388	.730	0.012	0.07	0.15
6^1D	4144	.440	0.011	0.13	0.31
7^1D	4009	.315	....	....	....
8^1D	3926	.305	....	....	....
9^1D	3870	.340	....	....	....
2^3P-5^3S	4120	.120	0.26	....	....
2^1S-4^1P	3964	.400	0.34	2.5	7.4
5^1P	3613	0.135	0.09	1.2	4.2

Suppose that the expanding shell has a thickness H and an average radius r. Then the total energy, in ergs, radiated by the entire shell in one second, in all directions, is

$$4\pi r^2 \, HN_n \, h\nu_{2n} \, A_{n2} \, .$$

We have measured this radiation in terms of the radiation of the continuous star spectrum. This is

$$\pi R^2 c \rho_\nu d\nu \, ,$$

where

$$\rho_\nu = \frac{8\pi h}{c^3} \frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \, .$$

We choose for $d\nu$ the frequency interval $\delta\nu$, corresponding to 1 A at the wave length considered. The quantity listed in the fourth column of Table 7 may be designated as x. Then

$$4\pi r^2 N_n H \, h\nu_{2n} \, A_{n2} = x\pi R^2 c\rho_\nu \, \delta\nu \, .$$

Recalling that, approximately,

$$W = \frac{R^2}{4r^2} ,$$

we have

$$\frac{1}{W} HN_n = \frac{xc\rho_\nu\delta\nu}{h\nu_{2n}A_{n2}} .$$

The values of A_{n2} are given by Goldberg.[14] Hence we may compute $(1/W)HN_n$; these quantities are given in the fifth column of Table 7, where we have used $T = 20{,}000°$. This choice must be explained. The energy from the nebula and from the unit in which we have measured it is weakened in the same proportion by interstellar absorption. Hence, in converting our unit into ergs per second, in all directions, we must use the amount of radiation within $\delta\nu$ as seen from a representative point within the nebula. Sherman and Morgan[17] have shown that "a considerable part of the observed reddening is probably due to physical conditions in the immediate vicinity of the star." It is impossible to ascertain whether the light of the continuous spectrum is already weakened when it reaches the nebula.

If it is, we should use $T < 20{,}000°$. This would greatly affect the unit but would not materially change the Boltzmann corrections. We should remember that for $T = 10{,}000°$ our unit would be almost ten times smaller and that all values of the last two columns of Tables 7 and 8 would be ten times larger.

The weaker absorption lines give the following quantity:

$$A_\lambda = \frac{\pi e^2}{mc^2} \lambda^2 HN_2 f_{2n} .$$

[17] *Ap. J.*, **89**, 516, 1939.

We may now compare the values of N_n derived from the emission lines with those of N_2 derived from the absorption lines. We may write

$$\frac{N_n}{N_2} = \frac{g_n b_n}{g_2 b_2} e^{-\left(\frac{h\nu_{2n}}{kT}\right)} ,$$

where b_n is Menzel's[18] factor, which measures the departure from thermodynamic equilibrium. This quantity is not known for helium. The last column of Table 7 gives the values of

$$\frac{\frac{1}{W} H N_n}{g_n e^{\frac{-h\nu_{1n}}{kT}}} .$$

These values should be compared with the corresponding quantity obtained from the weak absorption lines:

$$\frac{H N_2}{g_2 e^{\frac{-h\nu_{12}}{kT}}} .$$

The ratio of these quantities gives

$$\frac{1}{W} \cdot \frac{b_n}{b_2} .$$

Consider, for the diffuse triplets, the line λ 3554. The equivalent width is $A_\lambda = 0.2$ A. Hence,

$$N_2 H = 4 \times 10^{15} ,$$

and

$$\frac{N_2 H}{g_2 e^{\frac{-h\nu_{12}}{kT}}} = 8 \times 10^{19} .$$

Table 7 gives the results obtained from the emission lines.

Since the capture cross-sections for the various series of *He* I are not known, and since we do not even know to what extent recom-

[18] *Ap. J.*, **88**, 60, 1938.

bination is responsible for filling up the various levels, we cannot determine b_n/b_2. The tabular values show that the singlet D levels are relatively less populated than the triplet D levels; and since the differences in the statistical weights have already been taken into account, we must attribute the effect to differences in the values of b_n.

Hydrogen.—The Balmer series is followed in absorption to λ 3673.76 (2–23) and in emission to λ 3697.15 (2–17). Microphotom-

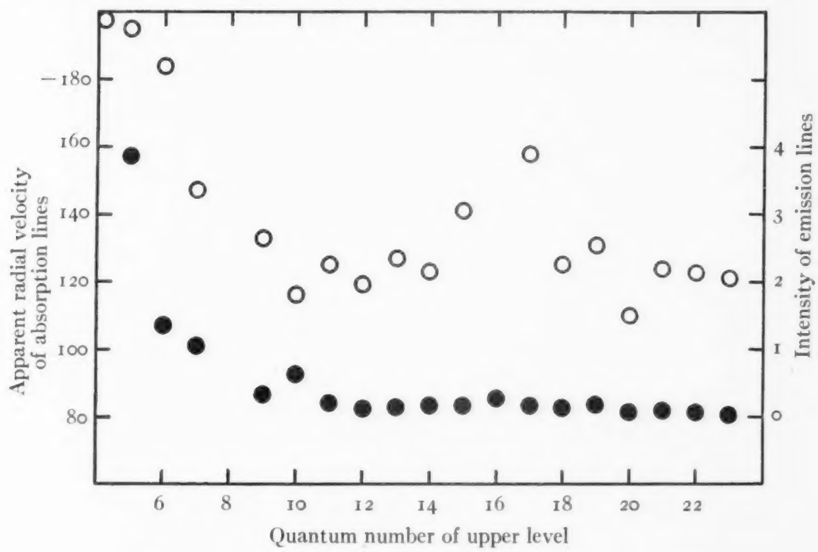


FIG. 3.—Radial velocities of the absorption lines of hydrogen
● Emission lines ○ Absorption lines

eter tracings strongly suggest that the emission lines also persist to the twenty-third member, but the eye does not distinguish the emission wings after number 17. In Figure 3 the apparent radial velocity of the absorption lines of hydrogen is plotted against the serial number of the line. The parallel plot of the emission line intensities indicates that the change in apparent velocity is caused by the relatively greater encroachment of the emission line on the violet absorption line for the early members of the series. This causes the absorption lines to be measured with systematically

shorter wave lengths and thus leads to a higher apparent negative velocity.

The persistence of the Balmer series to number 23 is evidence of high luminosity, as pointed out by Unsöld.[19] From Unsöld's calibration we find for the absolute magnitude about -5. In a recent paper Sherman and Morgan[17] found for P Cygni $M_{vis} = -4.4$.

TABLE 8

BALMER SERIES INTENSITIES

n (Upper)	E.W.	Emission Intensity	$(1/W)N_nH$ $(\times 10^{-15})$	$\frac{(1/W)N_nH}{2n^2e^{-\chi/kT}}$ $(\times 10^{-16})$
5	0.820	3.88	0.8	3.0
6	0.915	1.33	0.8	2.2
7	0.910	1.04	1.4	3.2
8	1.100	2.72	*	*
9	0.775	0.32	1.7	2.4
10	0.510	0.64	6.0	7.2
11	0.440	0.20	3.0	3.1
12	0.400	0.10	2.4	2.1
13	0.390	0.13	4.6	3.4
14	0.200	0.16	8.4	5.3
15	0.200	0.16	12.0	6.6
16	0.360	0.28	29.4	14.4
17	0.140	0.17	24.0	10.4
18	0.060	0.13	23.9	9.3
19	0.070	0.18	44.5	15.5
20	0.020	0.06	19.3	6.1
21	0.030	0.10	41.1	12.2
22	0.010	0.03	15.6	4.1
23	0.030	0.01	6.5	1.6

* Blended with *He* I 3888.64.

The measurement of the intensity of an absorption line flanked by an emission line, or vice versa, is complicated by the distortion of one by the other. The values of the equivalent widths of the absorption lines and the intensities of the emission lines, recorded in Table 8, second and third columns, were obtained by the measurement of the undistorted half of each line. We assumed that the true line is symmetrical. The fourth column gives, for the emission lines, the quantity $(1/W)N_rH$, and the fifth column gives the quantity

[19] Unpublished.

$[(1/W)N_rH/2n^2e^{-h\nu/\kappa T}]$. There is a tendency for this quantity to increase from $n = 5$ to $n = 19$. The decrease from $n = 19$ to $n = 23$ may not be real, because of the uncertainty in drawing the continuous spectrum and because of the weakness of the lines. The increase from $n = 5$ to $n = 19$ may be considered as an effect of the factor b_n of Menzel. From the observations $b_{20}/b_5 = 5$. The tables by Baker and Menzel[18] show that for $T_e = 20{,}000°$ neither their hypothesis *A* nor their hypothesis *B* provides for a ratio greater than about 3. A lower electron temperature, of the order of $T_e = 10{,}000°$, would be more consistent with the observations. It is premature to discuss this matter further.

TABLE 9

N_2H FOR HYDROGEN

n (Upper)	E.W.	log NHf	log f*	log NH	NH
20	0.020	11.22	−3.35	14.57	3.7×10^{14}
21	.030	11.40	3.42	14.72	5.2
22	.010	10.90	3.48	14.40	2.5
23	0.030	11.40	−3.54	14.94	8.7×10^{14}

* From Menzel and Pekeris, *M.N.*, **96**, 82, 1935.

Table 9 gives the results for N_2H from four weak *H* absorption lines. The mean is $N_2H = 5 \times 10^{14}$; $N_2H/2n^2e^{-h\nu/kT} = 2.3 \times 10^{16}$. This is almost identical with the value of $(1/W)N_nH/2n^2e^{-h\nu/kT}$ found for the emission lines having small values of n. Thus, the mean from Table 8 of the emission lines $n = 5$, $n = 6$, and $n = 7$ is 2.8×10^{16}. Hence,

$$\frac{1}{W}\,\frac{b_6}{b_2} = \frac{2.8 \times 10^{16}}{2.3 \times 10^{16}} = 1.2 \, .$$

From the tables of Menzel and Baker[18] we infer that b_6/b_2 is sensitive to T_e, as well as to the mechanism involved; but probably a value of $b_6/b_2 = 3$ is not excessive. This would give

$$\frac{1}{W} = 0.4 \, ,$$

and this is, of course, not consistent with the limitation $W < 1$.

We conclude that the emission lines are in reality too weak to be consistent with the absorption lines. Several explanations are possible:

a) The unit of measurement of the emission lines may be in error by as much as a factor of 10. If we adopt $T = 10{,}000°$ instead of $T = 20{,}000°$, we obtain, roughly,

$$\frac{1}{W} \cdot \frac{b_6}{b_2} = 10 ,$$

so that, for $b_6/b_2 = 3$, we find $W = 0.3$.

b) The assumption of a uniform shell, without appreciable obscuration by the star and without overlapping of emission and absorption, may not be permissible. This assumption rested upon the evidence from *He* I, *C* II, and *Mg* II that $W = 0.05$.

c) The metastability of the 2s level of *H* may cause this level to be overpopulated. We are unable to test this hypothesis because we do not know enough about the lifetime of the 2s level.

It is clear that the difficulty could best be overcome by observing the Paschen absorption lines in P Cygni. Attention must also be called to the fact that, although the inconsistency between absorption and emission may be overcome in P Cygni, it will be even more serious in 17 Leporis. The emission lines in that star are very faint. They are visible only in $H\alpha$ and $H\beta$. The absorption lines, on the other hand, are almost as strong as in P Cygni. The dilution factors are fairly well ascertained in both stars.

N II.—The nitrogen lines present an interesting case of departures from thermodynamic equilibrium: λ 3437 appears in absorption only, while the entire multiplet λλ 4601–4643 shows strong emission and absorption.

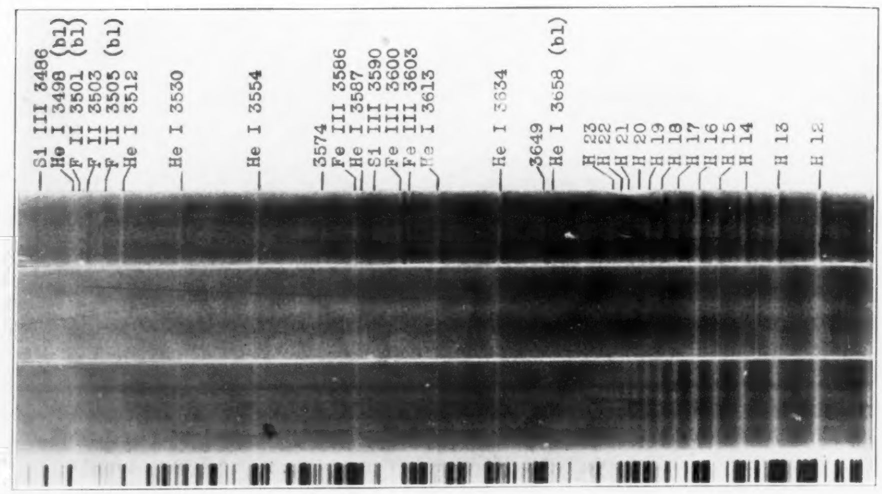
N III.—λ 3745.83 may be present, but the absence of λ 3754 and λ 3771 makes the identification doubtful.

O II.—Present: λ 3727 (blended); λ 3882 (1); λ 3470 (blended).

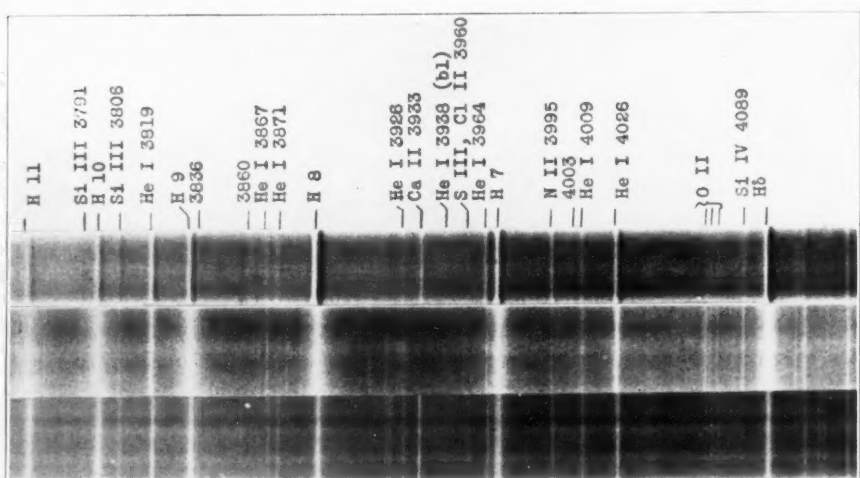
O III.—λ 3961 is absent.

F II.—The multiplet $3p^5P - 3d^5D°$ may be present, but two lines are blended with *Fe* III. The radial velocity of the third line is abnormal.

PLATE XVIII



a



b

SPECTRA OF P CYGNI (*top*), τ SCORPII (*middle*), AND 55 CYGNI (*bottom*)

Si III.—One of the most striking features of the spectrum of P Cygni is the peculiar behavior of the *Si* III lines. The triplet λλ 4552, 4567, and 4574 is in absorption with an emission wing on the red side of λ 4552, while the lines in the ultraviolet are pure emission lines with no evidence of any absorption. Table 10 summarizes the results.

TABLE 10

Si III IN P CYGNI

λ (Lab.)	E or A	Intensity	Velocity	Transition
3486.93	E	4	−44	Unclassified
3590.46	E	6	40	$4p^1P_1-4d^1D_2$
3791.41	E	2	18	$4p^3P_0-4d^3D_1$
3796.11	Blended with H_{10}		....	$4p^3P_1-4d^3D_1$
3806.56*	E	6	19	$4p^3P_2-4d^3D_1$
4552.608	A+E	8, 1	95 (−7)	$4s^3S_1-4p^3P_2$
4567.830	A	7	103	$4s^3S_1-4p^3P_1$
4574.750	A	5	−92	$4s^3S_1-4p^3P_0$

* Attention was called to the peculiar intensity of this line by Struve (*Ap. J.*, **81**, 95, 1935).

The Grotrian diagram for *Si* III (Fig. 4) indicates the relationship between the absorption triplet and the emission triplet. The 4^3P level is common to both. The interpretation of this departure from equilibrium may be possible when spectrograms are available for shorter wave lengths, to include the multiplets $3d^3D - 4p^3P^o$ (λ 3086, etc.) and $4p^3P^o - 5s^3S$ (λ 3230, etc.) as well as the singlet transition $4p^1P^o - 5s^1S$ (λ 3185). We note the fact that the $^3P^o$ level can reach the lowest triplet level only by two steps, a fact which may have something to do with the anomalous behavior of the two triplets.

S III.—λ 3324.85 is present. λ 3338.32 is blended with *N* II.

Fe III.—The *Fe* III lines are a marked feature of the spectrum. An examination of Plates XVII*b* and XVIII*a* shows quite clearly that the great strength of the *Fe* III lines is not a characteristic of the early B spectral classes, as these lines are not visible in τ Scorpii and 55 Cygni. In P Cygni they are, on the contrary, the strongest lines in the spectrum, with the exception of those due to hydrogen and helium.

The presence of *Fe* III in P Cygni was pointed out by Swings and

Struve.[20] The present study strengthens the identification. The *Fe* III spectrum has been analyzed by Swings and Edlén.[21]

In his paper on the photographic region of the spectrum of P Cygni, Struve[11] measured a strong line at λ 4396, which he tentatively identified as the *O* II line λ 4395.95. It is now much more probable

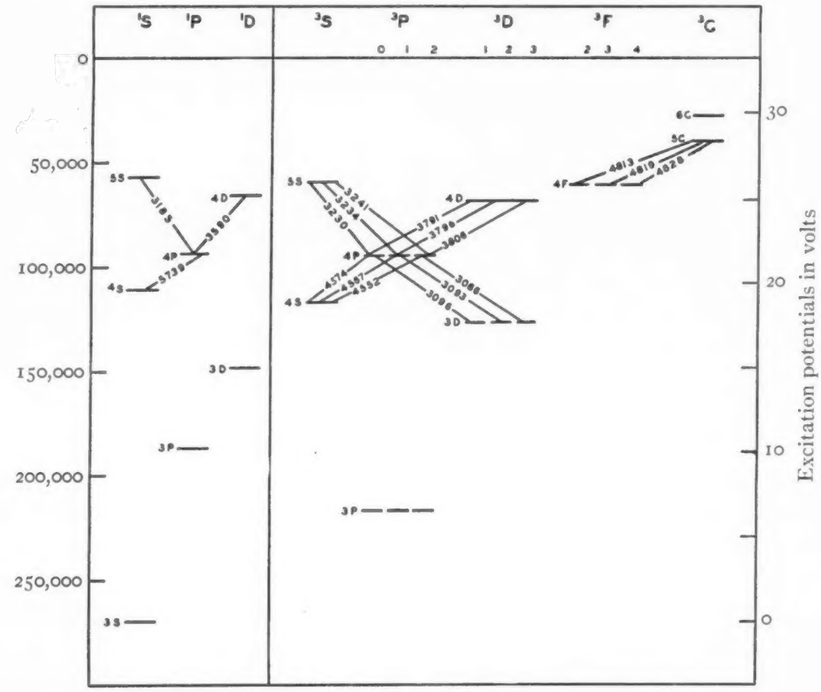


FIG. 4.—*Si* III: Energy-levels and spectral lines of astrophysical interest (from Bowen, *Phys. Rev.*, **39**, 8, 1932).

that the star line is identical with *Fe* III 4395.78. This removes the necessity of invoking abnormal fluorescence to explain the abnormal intensity of the *O* II line.

The mean radial velocity from the absorption lines of *Fe* III is −123 km/sec. This value agrees remarkably well with the correla-

[20] *Ap. J.*, **88**, 621, 1938.

[21] Dr. Swings has very kindly placed at our disposal, in advance of publication, the results of the analysis of *Fe* III by himself and Edlén.

tion found by Struve[11] between ionization potential and radial velocity. With its ionization potential of 30 volts *Fe* III should be similar to *He* I and *N* II. The ratio of emission to absorption for the *Fe* III lines agrees with the relation between this quantity and the ionization potential.[22]

Swings has called attention to the remarkable difference in the intensities of *Fe* III λ 4420 and λ 4164. These lines are almost identical in γ Pegasi, while in P Cygni λ 4420 is very much stronger. The latter starts from a metastable level, while λ 4164 starts from a normal level. We estimate that $W < 0.1$.

Although the strong lines of *Fe* III in the ultraviolet regions originate from levels which are not strictly metastable, it is clear that they must behave very much as though they were metastable. These lines originate from the levels a^5F and a^3F. The former can combine only with a single lower odd level, $4p^7P^o$. The line would be very weak, because of the change in multiplicity and the large change in *l*. Level a^3F can combine with two lower odd levels, $4p^7P^o$ and $4p^5P^o$. These intersystem combinations would also be weak. Hence the *Fe* III lines must be very strong for the same reason that λ 3832 of *Mg* I is strong in P Cygni.

Interstellar lines.—Two strong absorption lines are of interstellar origin: 3383.76 (*Ti* II) and 3933.67 (*Ca* II).

TABLE 11

UNIDENTIFIED LINES IN P CYGNI

λ (Star)	E or A	Intensity	λ (Star)	E or A	Intensity
3321.98	E	2	3699.37	A	1
3371.11	A	1s	3744.89	A	1
3405.50	E	2	3766.59	A	1s
3574.08	A	1	3957.48	A	1
3647.88	A	6	4003.16	A	3
3649.29	E	6			

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[22] *Ap. J.*, **81**, 87, 1935.

REVIEWS

Molekülspektren und Molekülstruktur, Part I: *Zweiatomige Moleküle.*[1] By Dr. GERHARD HERZBERG. Dresden and Leipzig: Theodor Steinkopff, 1939. Pp. xv+404. Figs. 169. Tables 37. Rm. 21; bound, Rm. 22.50.

This book concerns itself only with diatomic molecules. Part II will be devoted to polyatomic molecules and will appear as soon as possible. Part I is consistent in plan with Dr. Herzberg's earlier and well-received book, *Atomic Spectra and Atomic Structure*, and it should meet with equal favor.

The author has attained his objective of providing a suitable introduction for the beginner as well as a helpful compilation for the research worker. The style is lucid, and the observational material has been brought up to date. Both the empirical and the theoretical aspects of the problem are treated. In those instances where proofs are deemed too difficult for the beginner, an attempt is nevertheless made to provide him with an insight into the situation. He receives further assistance from the use of fine print in which are given the details of those portions of the book which may be omitted in an initial survey of the field. Good diagrams, figures, and tables are abundantly employed.

After a review of the fundamentals of atomic theory, an excellent presentation is made of the empirical relationships inherent in diatomic molecular spectra, covering the entire spectrum from the ultraviolet through the infrared. This is followed first by an adequate analysis of the rotation-vibration problem, made pointed by the use of examples from the literature. Electronic states and transitions are examined in the two succeeding chapters. The remaining chapters are devoted mainly to a discussion of the electronic configurations possible on theoretical grounds and

[1] [While this review went through the press, there appeared a revised translation of it under the title *Molecular Spectra and Molecular Structure* (Prentice-Hall, Inc., 1939; price, $6.50). The translation was made with the co-operation of the author by Professor J. W. T. Spinks of the University of Saskatchewan. Several additions and changes were made for the translation, and the table of molecular constants for the ground states of all diatomic molecules thus far investigated has been brought up to date as of July, 1939.—EDITORS.]

the bearing of this analysis upon heats of dissociation, dissociation processes and spectra, and valence. The book concludes with a chapter of examples, observational results, and applications to the fields of physics, chemistry, and astrophysics.

References to the literature are thorough, and the indexes are usefully done.

Arthur Adel

Lowell Observatory
Flagstaff, Arizona

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VOLUME 90 NUMBER 5

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